Polymer dynamics in random flow with mean shear

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Outline

- Motivation: Elastic turbulence
- Experimental setup
- Flow and polymer models
- Results:
    1. Angular statistics
    2. Polymer elongation distribution
- Conclusion

Elastic Turbulence

Elastic turbulence is recently discovered phenomena, where the chaotic fluid motion is observed in dilute polymer solutions at low Reynolds numbers.

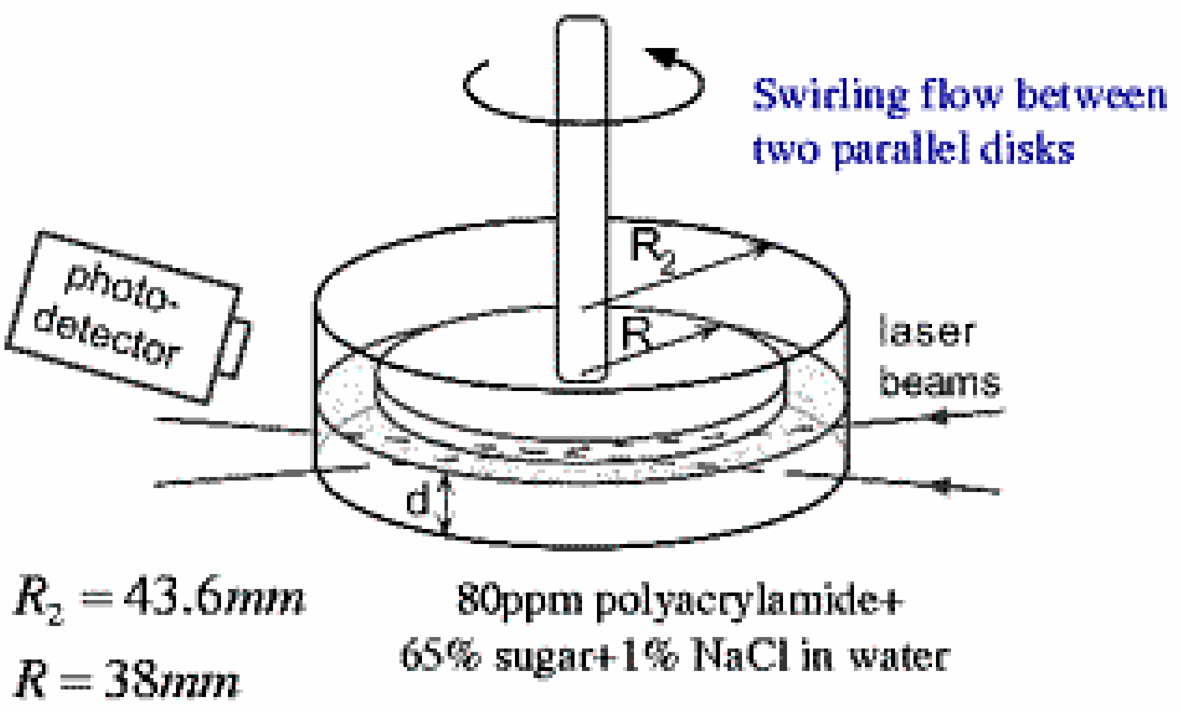
$$Wi \sim (\nabla u)\tau$$

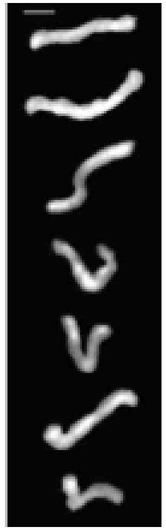
Well developed turbulence: $Re >> 1,\ Wi = 0$

Elastic turbulence: $Re << 1,\ Wi > 1$

A. Groisman, V. Steinberg, Nature **405**, 53 (2000);
Phys. Rev. Lett. **86**, 934 (2001)

Experimental Setup





Regular flow component is shear like: $V_r = V_z = 0, \quad V_\phi = \dfrac{\Omega r z}{d}$

Local shear rate is $s = \Omega r/d$

Polymer model

Polymer extension can be characterized by its end-to-end separation vector, ***R***, satisfying the following equation:

$$\partial_t R_i = R_j \nabla_j v_i - \gamma R_i + \zeta_i$$

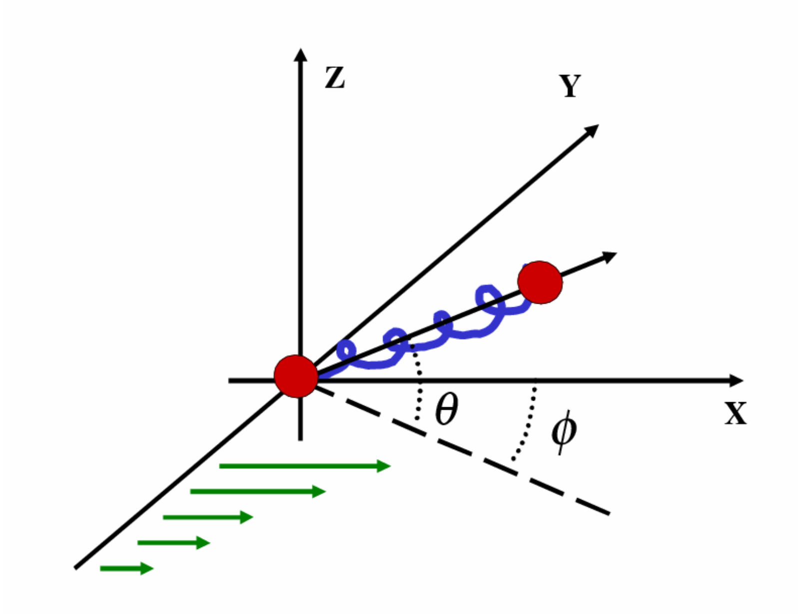
The relaxation rate is a function of polymer size $\gamma = \gamma(R)$

$$\gamma(R) = \gamma_0 / (1 - R^2 / R_{max}^2)$$

For stretched state the thermal noise ζ is negligible, and polymer orientation dynamics described by the unit vector **n**=**R**/R is completely decoupled:

$$\partial_t n_i = n_j (\delta_{il} - n_i n_l) \nabla_j v_l$$

Polymer orientation



The mean flow has the form $V_x = sy$, $V_y = V_z = 0$ and the Polymer orientation vector Is parameterized as

$$n_x = \cos\theta\cos\phi,\ n_y = \cos\theta\sin\phi,\ n_z = \sin\theta$$

Angular dynamics

Equation for ***n*** acquires the following form:

$$\partial_t \phi = -s \sin^2 \phi + \xi_\phi$$

$$\partial_t \theta = -s \frac{\sin(2\phi)}{2} \sin\theta \cos\theta + \xi_\theta$$

For isotropic short-correlated chaotic velocity component

$$\langle \xi_\theta(t) \xi_\theta(t') \rangle = 4D\delta(t-t')$$

$$\langle \xi_\phi(t) \xi_\phi(t') \rangle = \frac{4D}{\cos^2\theta} \delta(t-t')$$

For $s>>D$ one can introduce the characteristic angle $\phi_t \sim (D/s)^{1/3} << 1$ where the regular and stochastic terms are of the same order. The characteristic time is estimated as $\tau_t = 1/(s\phi_t)$. The dimensionless parameter $Wi \sim s/D >> 1$

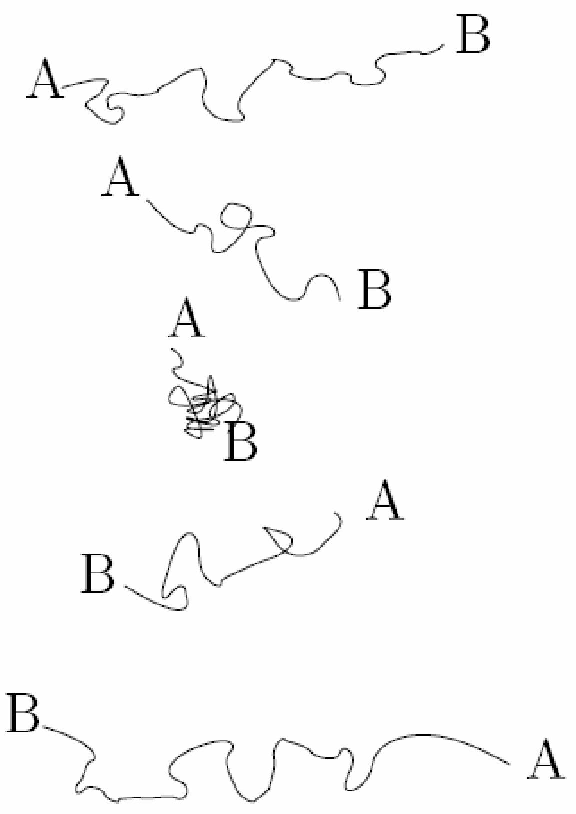
Qualitative picture

There are two different regions:

The region $|\phi| \sim \phi_t \sim Wi^{-1/3}$, where the polymer spends most of the time is stochastic because the dynamics is determined by the random velocity component.

For $|\phi| >> \phi_t$ the polymer is driven by the strong regular shear term.

Polymer tumbling



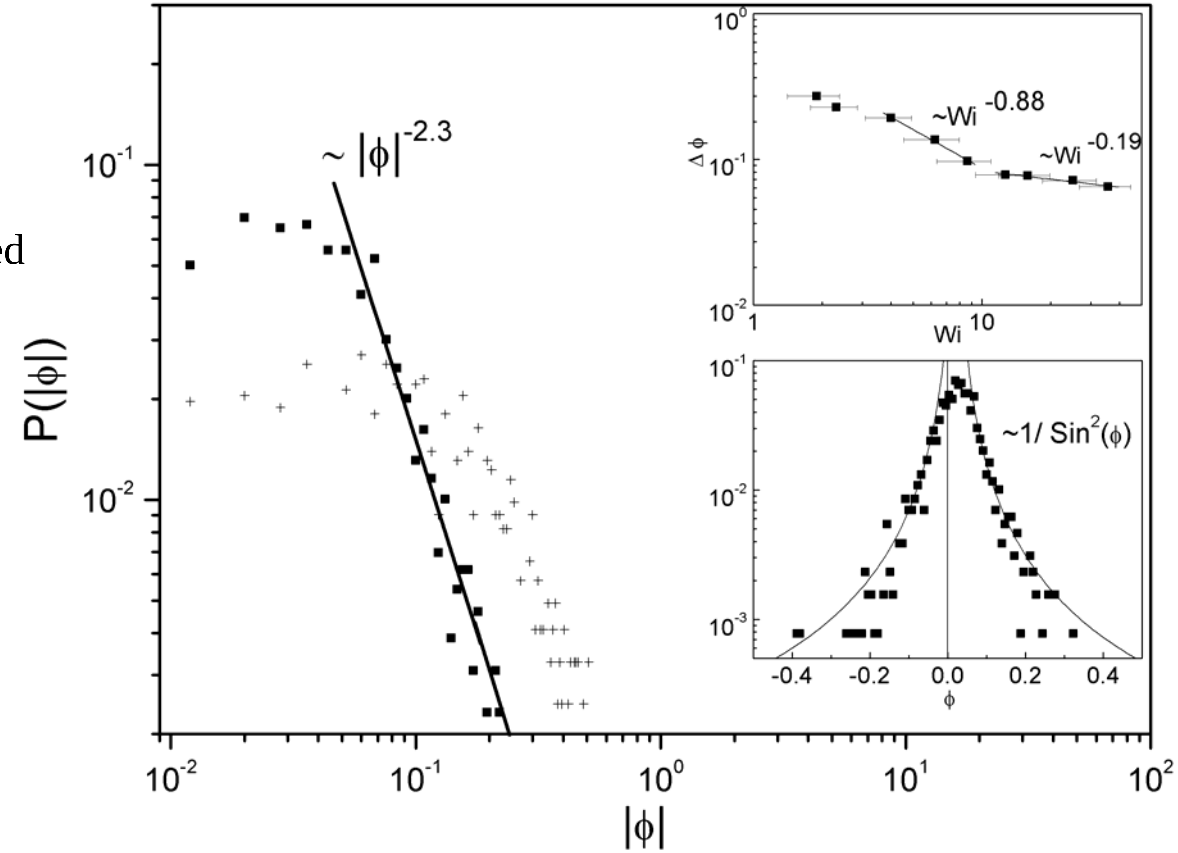
D. E. Smith and S. Chu, Science 281, 1335 (1998)
D. E. Smith, H. P. Babcock, and S. Chu, Science 283, 1724 (1999).

ϕ angle distribution

$$\partial_t \phi = -s \sin^2 \phi + \xi_\phi$$

The distribution of the angle ϕ is asymmetric, localized at the positive angles of order ϕ_t with the asymptotic $P(\phi) \sim sin^{-2}\,\phi$ at large angles determined by the regular shear dynamics.

$$\mathcal{P}(\phi, \theta) = \frac{U(\tan\theta/\sin\phi)}{\sin^3\phi\,\cos^2\theta}$$



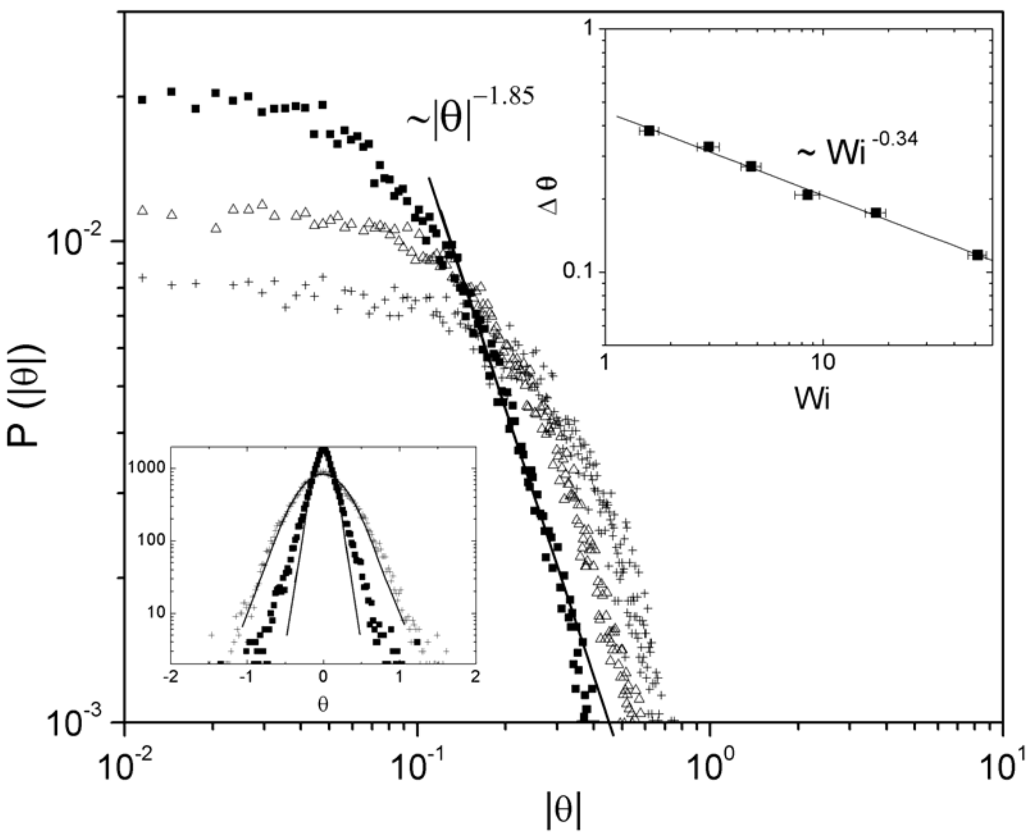
θ angle distribution

$$\partial_t\theta = -s\frac{\sin(2\phi)}{2}\sin\theta\cos\theta + \xi_\theta$$

PDF of the angle θ is also localized at $\theta \sim \phi_t$ with algebraic tails in intermediate region $\phi_t << |\theta| << 1$. These tails come from the two regions: the regular one gives the asymptotic $P(\theta) \sim \theta^{-2}$, and from the stochastic region, which gives a non-universal asymptotic $P(\theta) \sim \theta^{-x}$, where x is some constant which depends on the statistical properties of the chaotic velocity component.

$$P(\theta) \propto |\theta|^{-S'(x^*)}$$

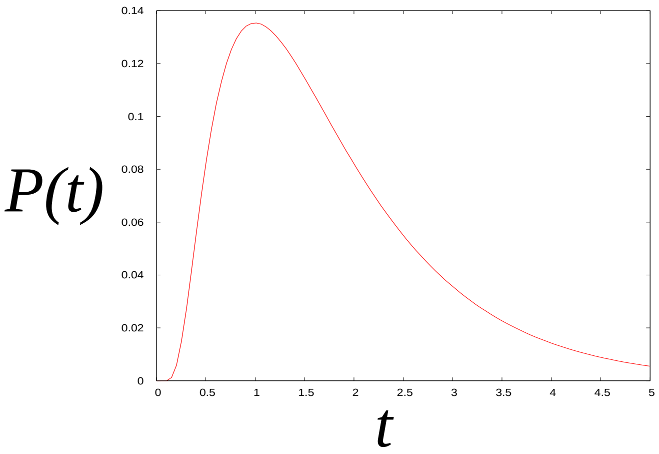
$$S(x^*) = x^* S'(x^*)$$

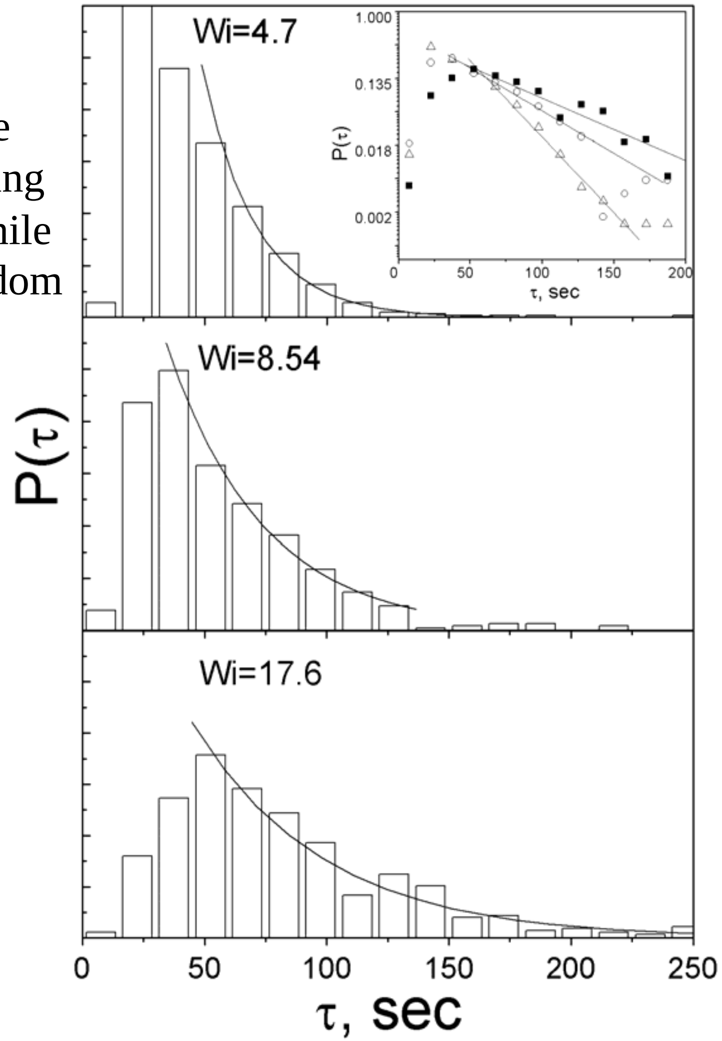


Tumbling time distribution

The characteristic tumbling times are of order $\tau_t=1/(s\phi_t)$, however due to stochastic nature of the tumbling process there are tails corresponding to the anomalous small or large tumbling times. The right tail always behaves like $P(t) \sim exp(-c\ t/\tau_t)$, while the left tail is non-universal depending on the statistics of random velocity field.

$$P_\tau = \frac{2\pi^2}{\tau^3 s} P_\xi \left(-\frac{\pi^2}{\tau^2 s}\right)$$





Polymer elongation distribution

Basic equation describing the polymer size dynamics:

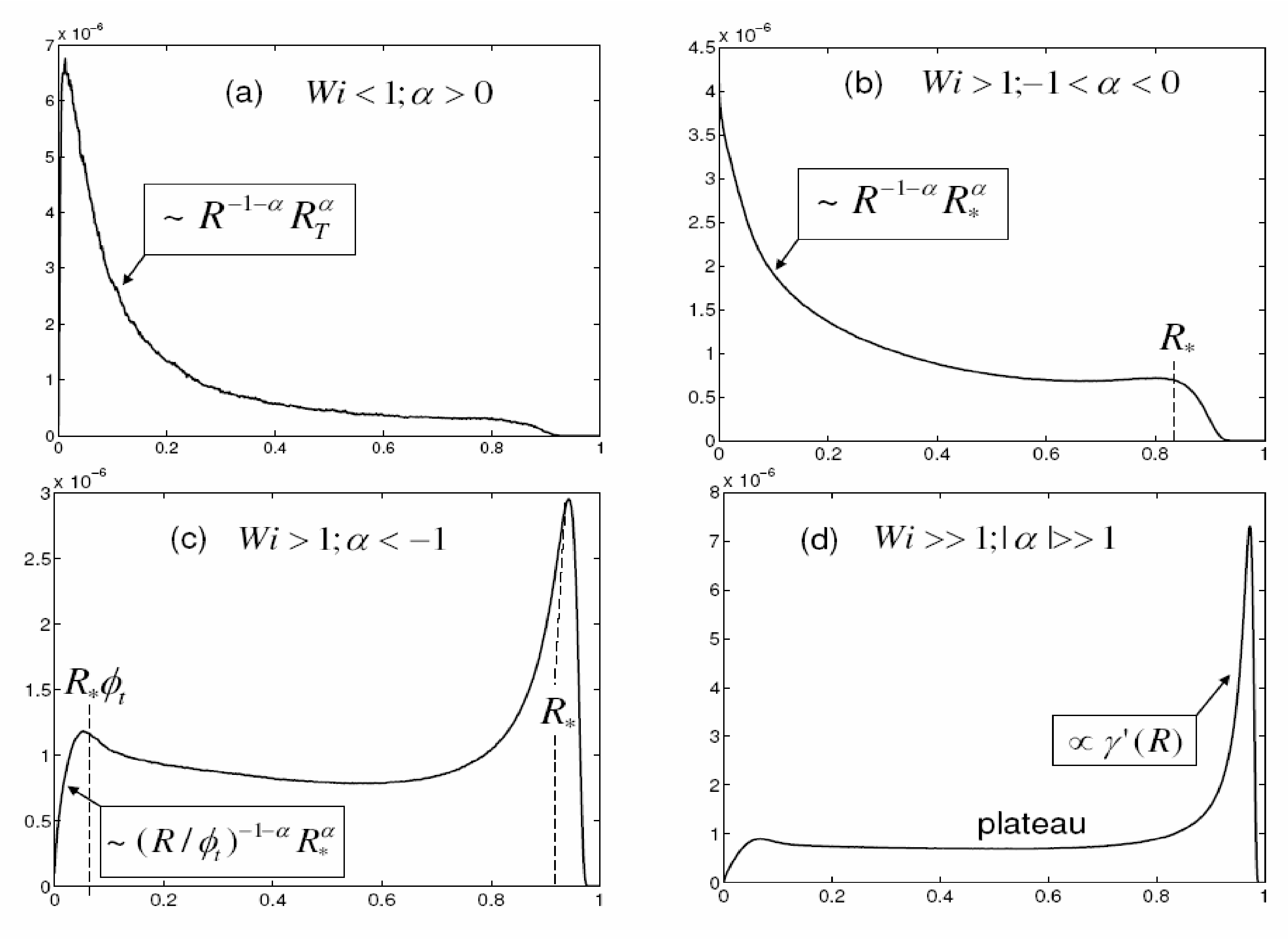
$$\partial_t R = (s \cos^2\theta \cos\phi \sin\phi - \gamma(R) + \xi)R + \zeta$$

We use the usual FENE-P model with the following relaxation rate: $\gamma(R) = \gamma_0/(1 - R^2/R^2_{max})$

The form of the elongation PDF crucially depends on the Weissenberg number which can be written as $Wi = \lambda/\gamma_0$

In the case $Wi >> 1$ the PDF will be centered at R_* which is found from the condition $\gamma(R^*) = \lambda$

Polymer elongation PDF



Results

- Stationary angular PDF
- Tumbling time distribution
- Polymer size distribution

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