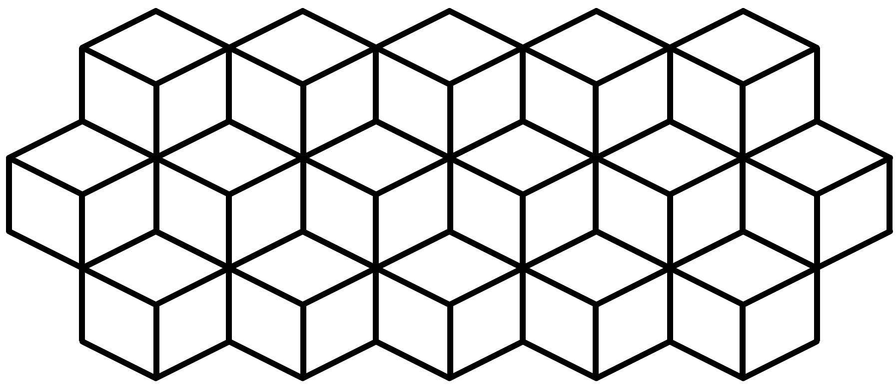
UNIFORMLY FRUSTRATED XY MODEL

$$H = -J \sum_{(\mathbf{jk})} \cos\left(\varphi_{\mathbf{k}} - \varphi_{\mathbf{j}} - A_{\mathbf{jk}}\right), \quad \Sigma\, A_{\mathbf{jk}} = 2\pi f$$

In terms of a Josephson junction array

$$A_{\mathbf{jk}} = \frac{2\pi}{\Phi_0} \int_{\mathbf{r_j}}^{\mathbf{r_k}} d\mathbf{r} \cdot \mathbf{A}(\mathbf{r}) \,, \qquad \Rightarrow \qquad f = \frac{\Phi}{\Phi_0} \,.$$

Dice lattice:

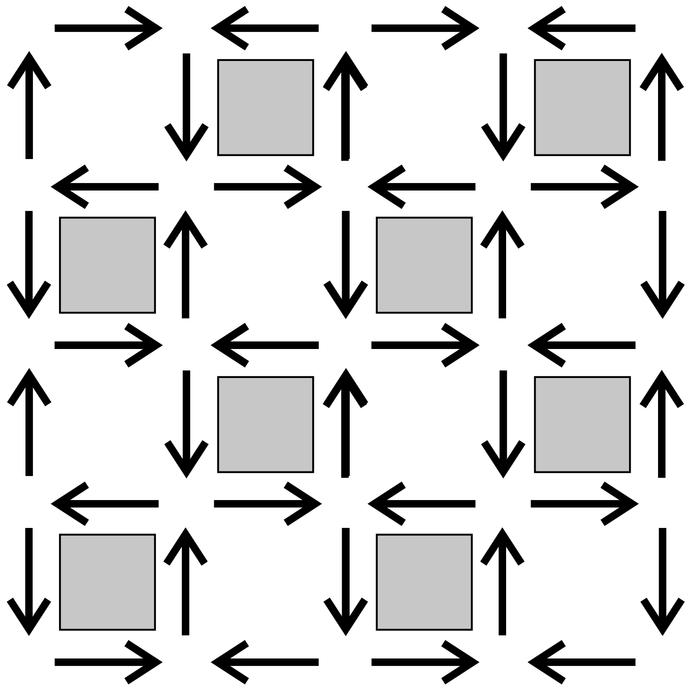


Gauge-invariant description:

$$\theta_{\mathbf{jk}} = \varphi_{\mathbf{k}} - \varphi_{\mathbf{j}} - A_{\mathbf{jk}} \qquad (-\pi < \theta_{\mathbf{jk}} < \pi)$$

$$\Sigma\,\theta_{\mathbf{jk}} = \begin{cases} +2\pi(1-f) \\ \\ -2\pi f \end{cases} \qquad = 2\pi m$$

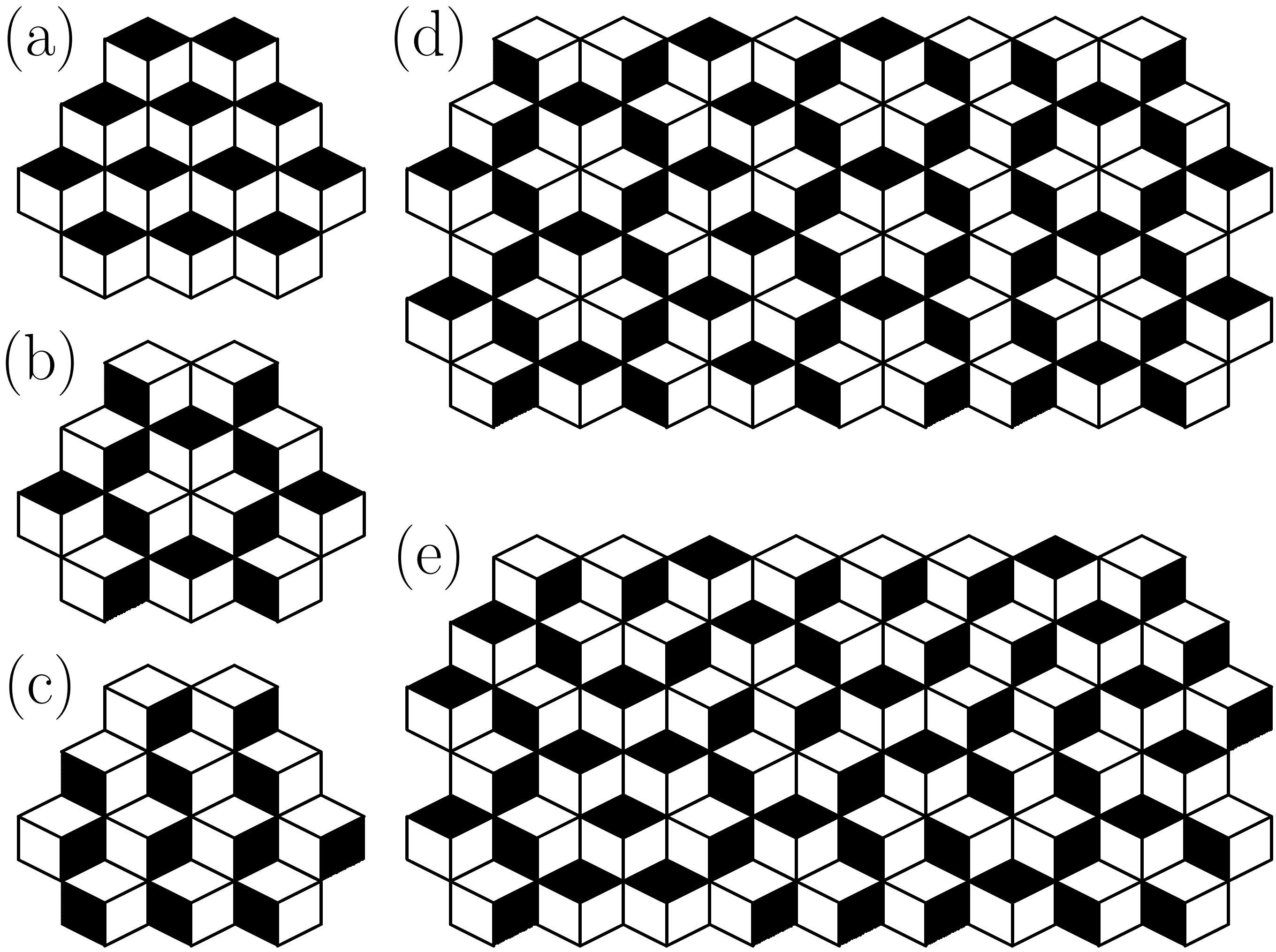
Square lattice, $\underline{f = 1/2}$ $\qquad m = \pm\frac{1}{2}$

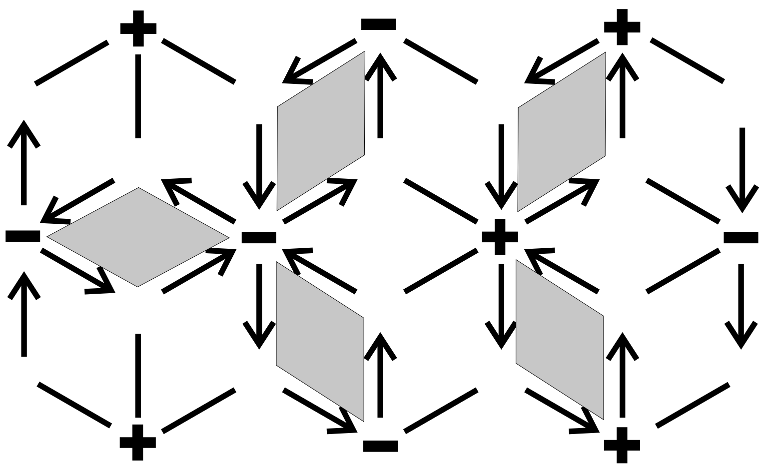


$$\theta_{\mathbf{jk}} = \pm\frac{\pi}{4}$$

Dice lattice, $\underline{f = 1/3}$

$$m = \begin{cases} +2/3 \\ -1/3 \end{cases}$$





$$\theta_{\mathbf{ij}} = 0\,,\ \pm\frac{\pi}{3}$$

The antiferromagnetic Ising model on triangular lattice:

- is disordered at any finite temperature

Houtappel, 1950

Wannier, 1950

- at $T = 0$ is characterized

  by algebraic correlations

Stephenson, 1964

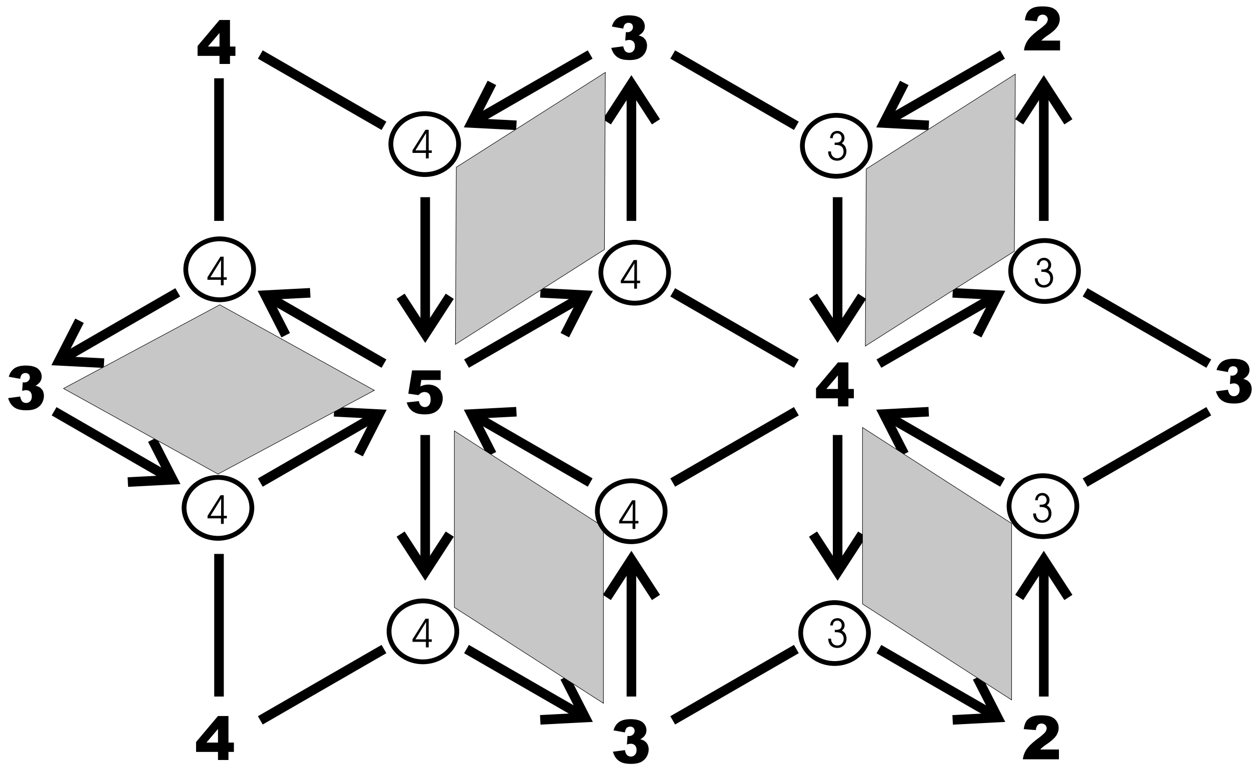
  and extensive residual entropy;

Wannier, 1950

and can be mapped onto a SOS model

Blöte and Hilhorst, 1982

SOS REPRESENTATION



$$h_{\mathbf{j}\pm\mathbf{e}_\alpha} = h_{\mathbf{j}} \pm 3m_{\mathbf{j},\mathbf{j}\pm\mathbf{e}_\alpha}, \quad n_{\mathbf{k}} = \frac{1}{3}\sum_{\mathbf{j}=\mathbf{j}(\mathbf{k})} h_{\mathbf{j}}$$

ZERO TEMPERATURE FLUCTUATIONS

$$\langle m_{\mathbf{j}_1\mathbf{j}_1'} m_{\mathbf{j}_2\mathbf{j}_2'} \rangle \propto |\mathbf{j}_1 - \mathbf{j}_2|^{-\eta} \,, \qquad \eta = 2$$

Stephenson, 1970

$$\langle (h_{\mathbf{j}_1} - h_{\mathbf{j}_2})^2 \rangle \propto \frac{9}{\pi^2} \ln |\mathbf{j}_1 - \mathbf{j}_2|$$

Blöte and Hilhorst, 1982

Accordingly, the large-scales fluctuations of h can be described by a continuous Hamiltonian,

$$H = \frac{K}{2} \int d^2\mathbf{r} \, (\nabla h)^2 \,,$$

where $K = K_0 = \pi/9$. The transition to the smooth phase would take place when $K = \pi/2$.

LOW TEMPERATURE FLUCTUATIONS

Comparison of the spin-wave free energies of the three periodic states (calculated in the harmonic approximation) gives

$$F_{\rm h} < F_{\rm s} = F_{\rm z}\,,$$

$$\delta F \approx \gamma T\,, \qquad \gamma \approx 2.27 \times 10^{-3}\,,$$

which in terms of h is translated into

$$K = K_0 + \sqrt{3}\gamma\,.$$

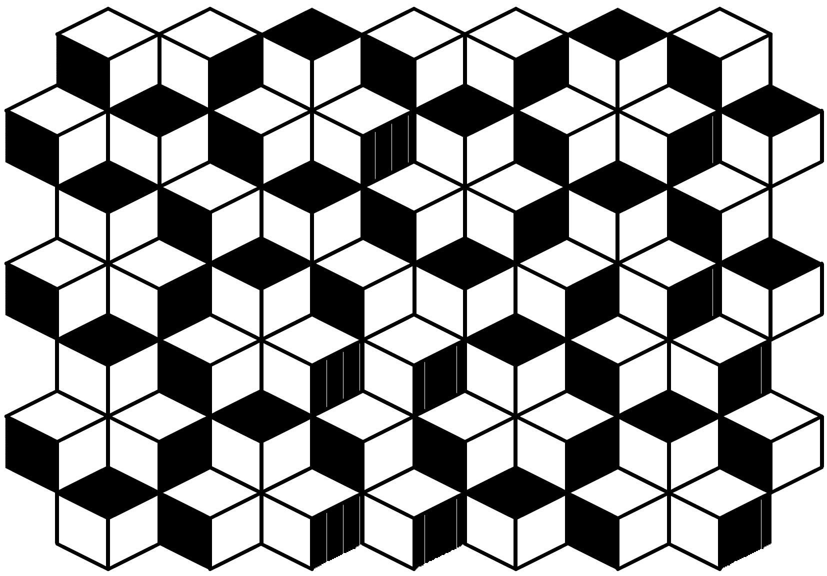
TOPOLOGICAL EXCITATIONS

Ordinary vortices.

On crossing a step the value of $\varphi_{\mathbf{j}}$ on 6-fold co-ordinated sites

$\varphi_{\mathbf{j}} \Rightarrow \varphi_{\mathbf{j}} + \pi$	on one out of three sublattices
$\varphi_{\mathbf{j}} \Rightarrow \varphi_{\mathbf{j}}$	on the other two

Three steps of the same sign can merge together forming a half-vortex, which simultaneously is a dislocation with Burgers number $b = \pm 3$:



The interaction of half-vortices:

$$V(\mathbf{R}_1, \mathbf{R}_2) = 2\pi\Gamma_0 \ln|\mathbf{R}_1 - \mathbf{R}_2| M_{\mathbf{R}_1} M_{\mathbf{R}_2} + \frac{KT}{2\pi} \ln|\mathbf{R}_1 - \mathbf{R}_2|\, b_{\mathbf{R}_1}\, b_{\mathbf{R}_2}$$

$$\Gamma_0 = \frac{5}{4\sqrt{3}} J\,, \quad M_{\mathbf{R}_{1,2}} = \pm\frac{1}{2}\,, \quad b_{\mathbf{R}_{1,2}} = \pm 3\,.$$

Dislocations. A pair of half-vortices of the opposite signs can form a dislocation with $b = \pm 6$. The interaction of such dislocations is too weak to keep them bound in pairs. This leads to the disordering of vortex pattern at any T.

Unbinding of half-vortex pairs:

$$T = \frac{\pi}{8}\Gamma(T) < \frac{\pi}{8}\Gamma_0 \approx 0.28\, J$$

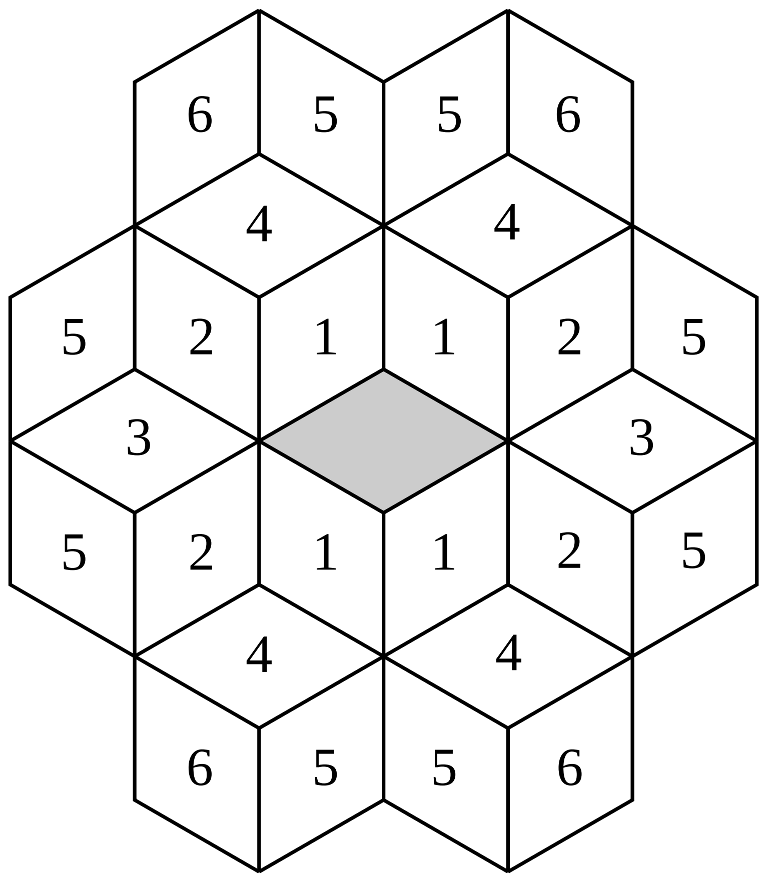
.

Magnetic interaction of currents

makes the step energy finite:

$$E_{\rm st} = -\frac{\mu J^2}{E_\Phi}\,, \quad E_\Phi = \frac{\Phi_0^2}{4\pi^2 a}$$

For $a \approx 8\,\mu{\rm m}$ $E_\Phi \approx 0.98\cdot 10^4\,{\rm K}$, whereas $\mu \approx \sin^2(\pi/3)\cdot[\lambda_2 - \lambda_4 - 2(\lambda_5 - \lambda_6) + \ldots] \approx 0.25$, $\lambda_i \equiv -L_i/a > 0$.



At $T \ll |E_{\rm st}|$ the striped state will be stabilized. From the comparison with numerical simulations of the Ising model with NNN interaction,

$$T_c \approx 3|E_{\rm st}| \ , \quad \tau_c \equiv \frac{T_c}{J} \approx \sqrt{\frac{3\mu T}{E_\Phi}} \approx 0.02 \ .$$

However, the observation of such an ordering is likely to be impossible for dynamic reasons. The "neighboring" ground states are separated by the barriers with $E_{\rm b} = 6(2-\sqrt{3})J \approx 1.61\,J$, so

$$\exp\left(-\frac{E_{\rm barr}}{T}\right) \sim \begin{cases} 10^{-14} & \text{for } T/J = 0.05 \\ 10^{-34} & \text{for } T/J = 0.02 \end{cases}$$