Conductivity of granular metal

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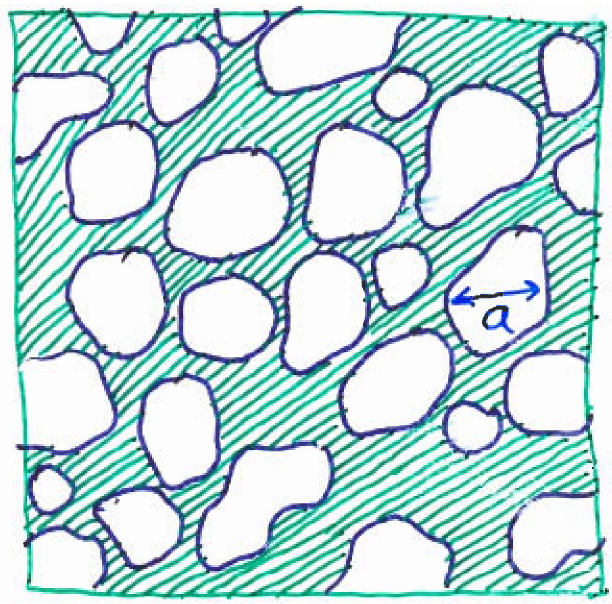
Granular Metal



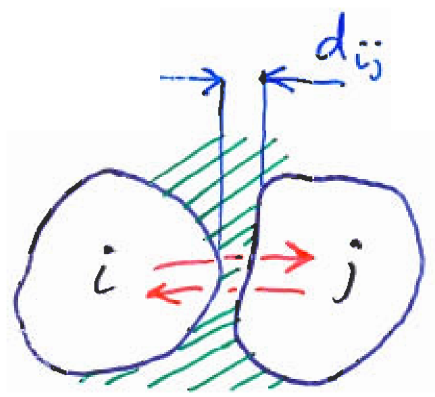
Metal grains

in an

Insulating

Matrix

$a \sim 100 - 1000\,\text{Å}$



Intergrain tunneling

Dimensionless conductance:

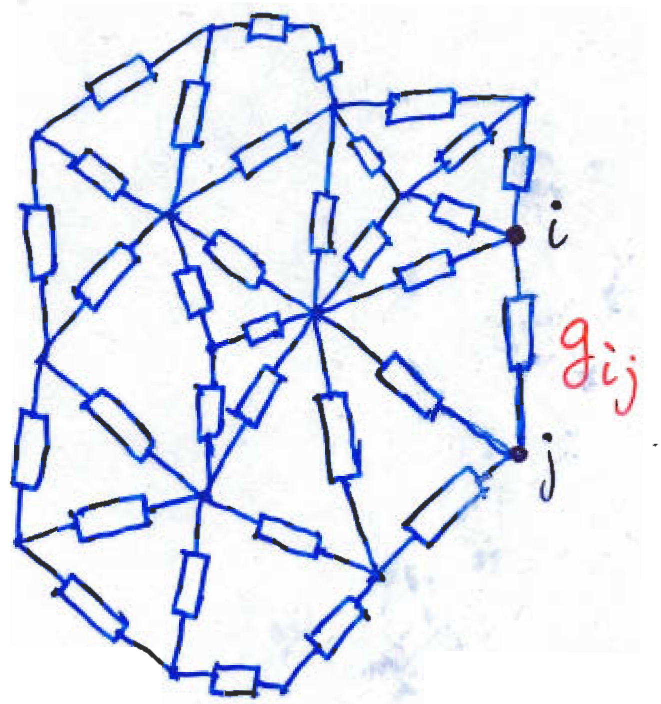
$$\boxed{g_{ij} \equiv \frac{h}{e^2 R_{ij}}}$$

$$g_{ij} \propto \exp(-2\varkappa \underline{\underline{d_{ij}}})$$

Strong fluctuations!

Macroscopic conductivity?

Conductivity σ: naive model I



Random resistor network

$$\sigma \sim \langle g \rangle_{\text{properly averaged}} \rightarrow T\text{-independent}$$

Percolation theory etc, see Kirkpatrick '70

This model only works at very high Temperature

Experiments: considerable T-dependence up to room temperatures!

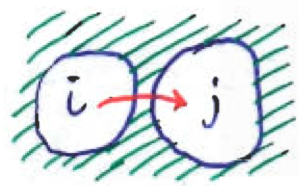
Electrostatic effects

$$H_{Coulomb} = \sum_{ij} \Delta N_i \Delta N_j E_{ij}^{(c)} + \sum_i \Delta N_i V_i$$

- ΔN_i – excess charge on i^{th} grain
- $E_{ij}^{(c)} = \frac{e^2}{2} [\hat{C}^{-1}]_{ij}$ – inverse capacitance matrix
- V_i – random potentials (due to stray charges in the insulator)

$\boxed{\text{For } a \sim 100-200\,\text{Å}}$ $\boxed{E_c \sim \frac{e^2}{\epsilon a} \sim 500-1000K}$

Coulomb Blocade !



$\Delta E_{ij} \sim E_c$

Conductivity : naive model II

Random resistor network with

$$g_{ij} \longrightarrow \tilde{g}_{ij} = g_{ij}\, e^{-\frac{\Delta E_{ij}}{T}}$$

$$\Delta E_{ij} = E_{ii}^{(c)} + E_{jj}^{(c)} - 2E_{ij}^{(c)} + V_j - V_j > 0$$

$$\sigma_0 \to \sigma = \sigma_0 \exp\{-E_A/T\}, \quad E_A \sim E_c$$

$\boxed{\text{Only works at high } T \text{ and } g \ll 1\,!}$

Conduction in granular aluminum near the metal-insulator transition

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(Received 3 March 1980)

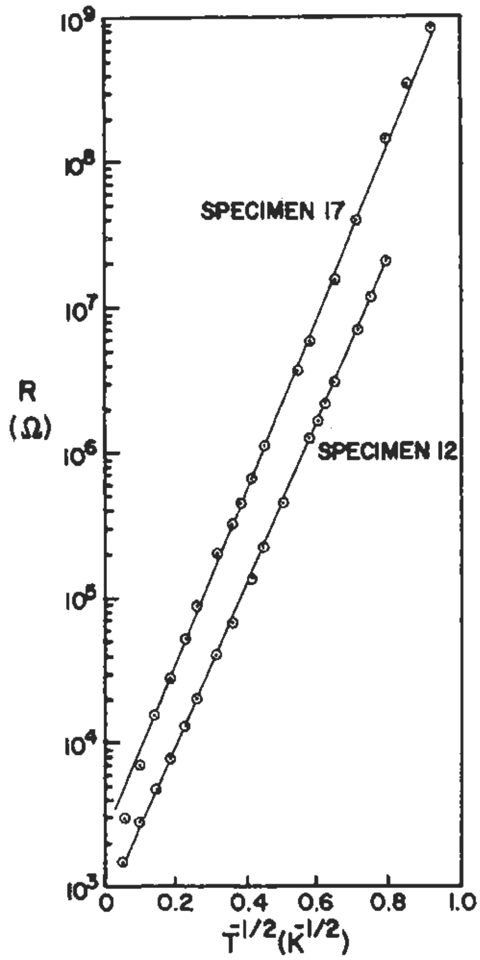

FIG. 2. Log$R$ as a function of $T^{-1/2}$ for the insulating, nonsuperconducting specimens, 12 and 17. (The measured resistance of specimen 17 has been multiplied by a factor of 9.3. This normalizes the graphs to the same geometrical factor; i.e., it makes the ratio of the plotted resistances for the two specimens equal to the ratio of their resistivities.)

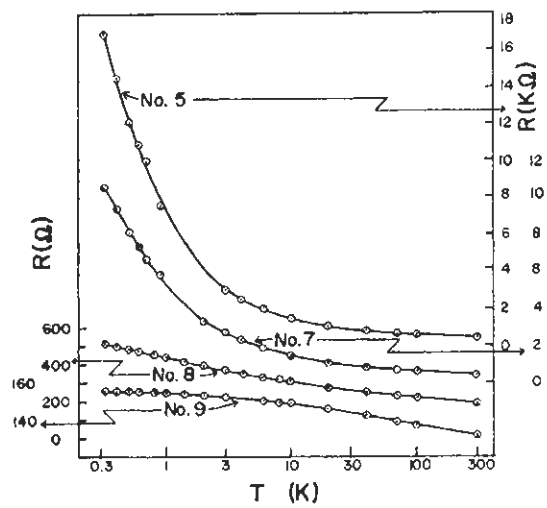

FIG. 3. Normal-state resistance as a function of log$T$ for the metallic specimen, 9, and specimens 8, 7, and 5, which are in the intermediate regime.

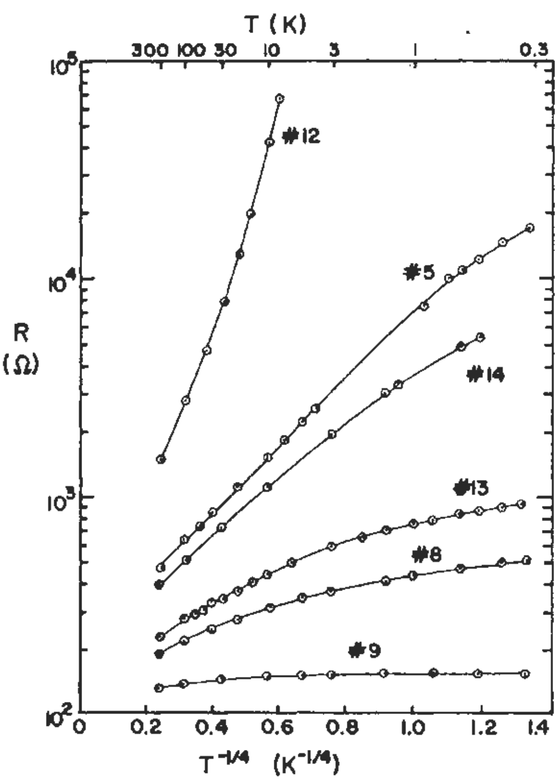

FIG. 4. Log$R$ as a function of $T^{-1/4}$ for six specimens. Specimen 9 is in the metallic regime, specimen 12 is in the insulating, strong-localization regime, and the others are in the intermediate, weak-localization regime.

high density of states. The implication is, however, that at sufficiently low temperatures all disordered semiconductors will exhibit correlation effects, so that the limiting behavior as T goes to zero will never be that of the Mott hopping law.[4]

The characteristics of our specimens are given in Table I. They were made by evaporating pure aluminum in the presence of a small amount of oxygen. The aluminum then forms metallic grains surrounded by amorphous aluminum oxide. By varying the oxygen pressure the resistivity can be changed over a wide range, from the metallic to the insulating regimes. We report here on specimens with the values of ρ_{RT} from 1.5×10^{-3} to 1.3×10^{-1} Ω cm. In this region the grain size is known to be rather uniform,

2a

Insulator-Superconductor Transition in 3D Granular Al-Ge Films

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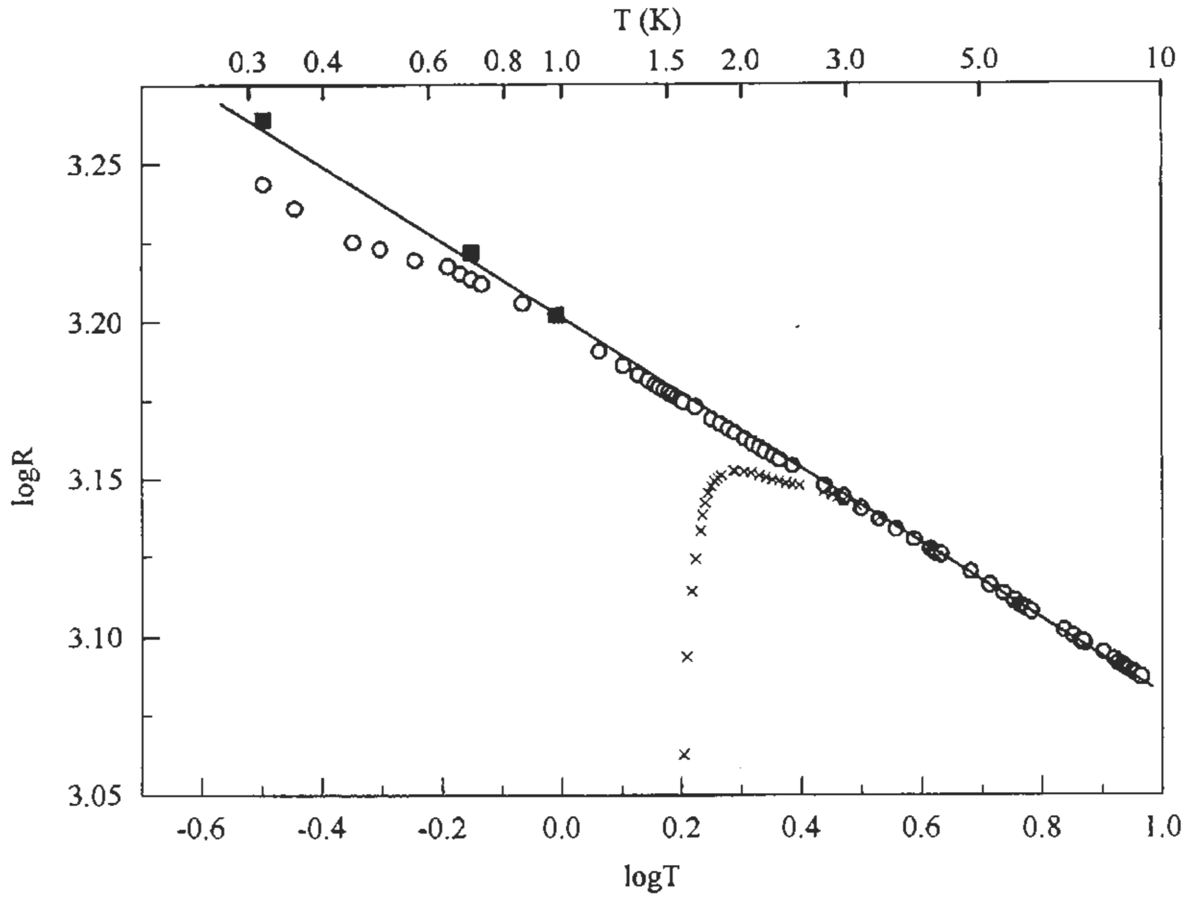

FIG. 3. Resistance of sample 3 as a function of temperature on a log-log scale, as measured at (zero) (×) and 100 kOe field (open circles). Open circles indicate resistance measured with a constant dc current $I = 10^{-5}$ A. Solid squares are zero bias resistances approximated from I-V measurements. Sample 3 room temperature resistance is 500 Ω.

2c

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Transport measurements in granular niobium nitride cermet films

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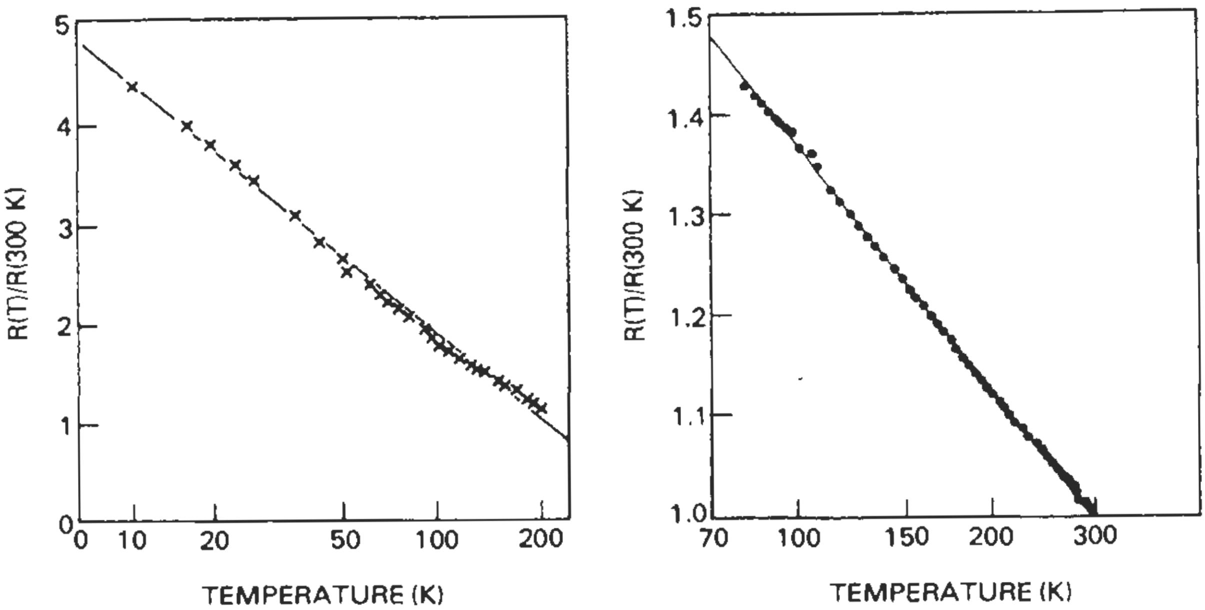

FIG. 5. The resistance normalized to the room-temperature value vs $\ln T$ for two different superconducting samples. The graph on the left is for a sample with $R_{\square}(300\ \mathrm{K})=2000\ \Omega/\square$, while that on the right is for $R_{\square}(300\ \mathrm{K})=100\ \Omega/\square$. The lines are guides to the eye.

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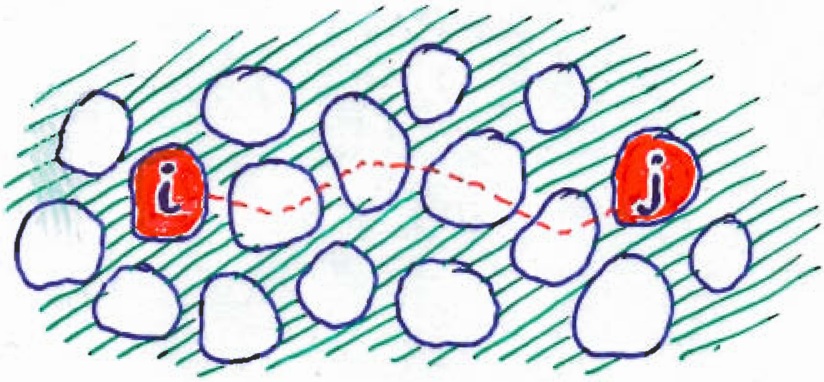
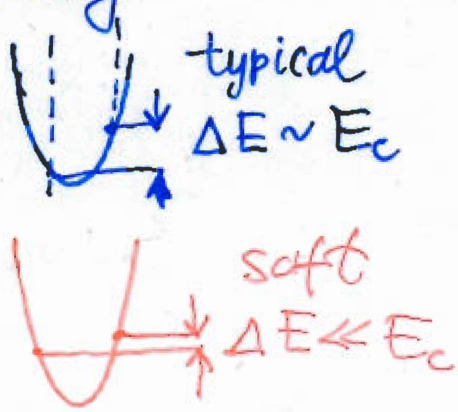
Additional Effects

- Randomness + distant cotunneling



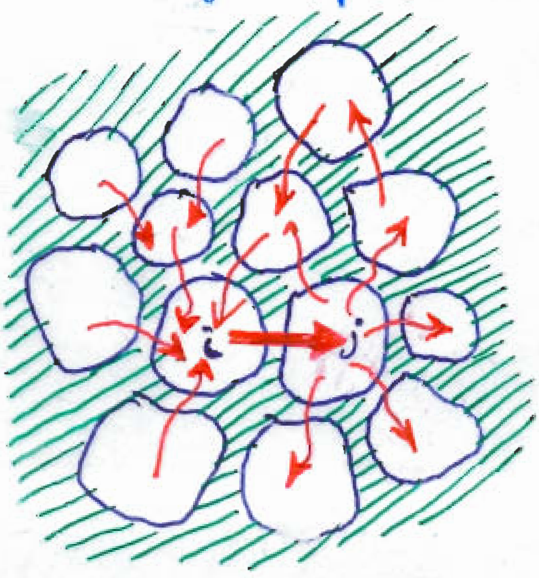


For certain distant pairs $i\, j$ ΔE_{ij} may be small!

Variable range cotunneling $\boxed{\sigma \propto e^{-(T_{ES}/T)^{1/2}}}$

Efros - Shklovskii law $\boxed{T_{ES} - ?}$

- Charge spreading (for $g \gg 1$)



* Transition time

$\tau_{tr} \sim \hbar / E_c$

* Relaxation time

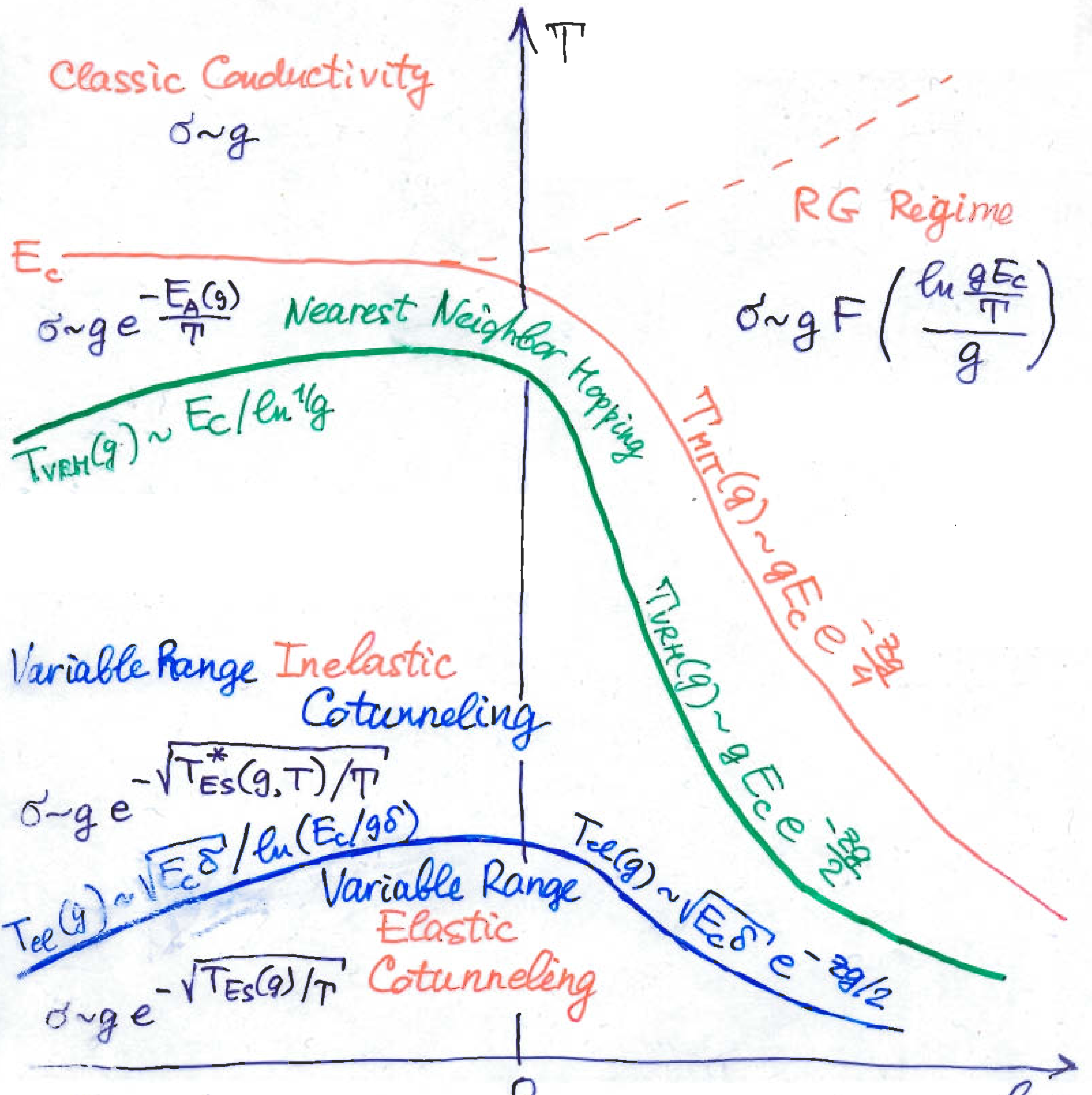
$\tau_R \sim RC \sim \hbar / g E_c$

$\boxed{\tau_R \ll \tau_{tr}}$ for $g \gg 1$

Compensating currents suppress the Coulomb Blockade.

$$\boxed{\tilde{E}_c \sim E_c\, e^{-\tau_{tr}/\tau_R} \sim E_c\, e^{-gz/2}}$$

Overall Phase Diagram

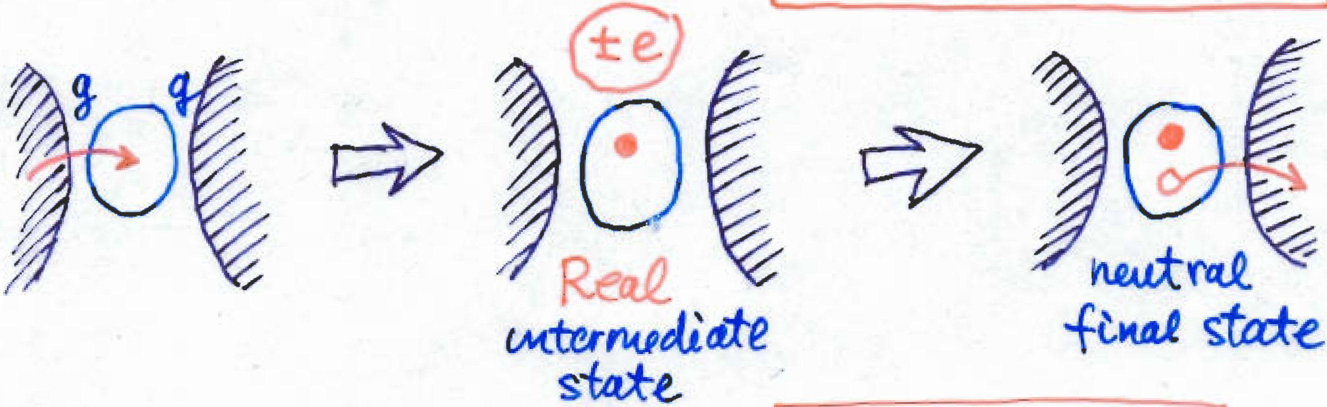


Parameters:

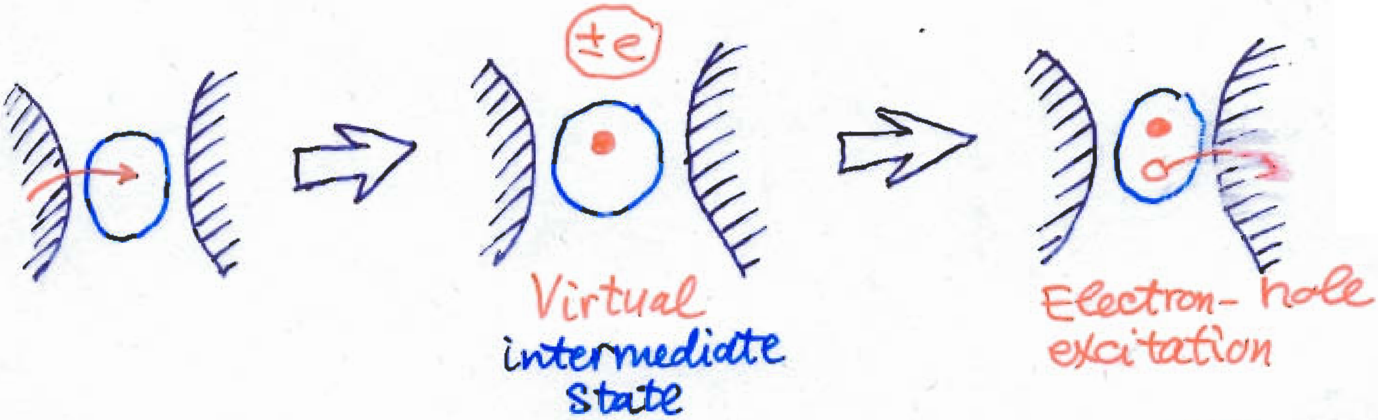
- T - temperature
- $E_c \sim 300 - 1000\,K$ Coulomb energy
- $\delta \sim 0.1\,K$ Characteristic level spacing in grains
- $g \lesssim 1$ Characterictic dimensionless conductance

Cotunneling : Standard description

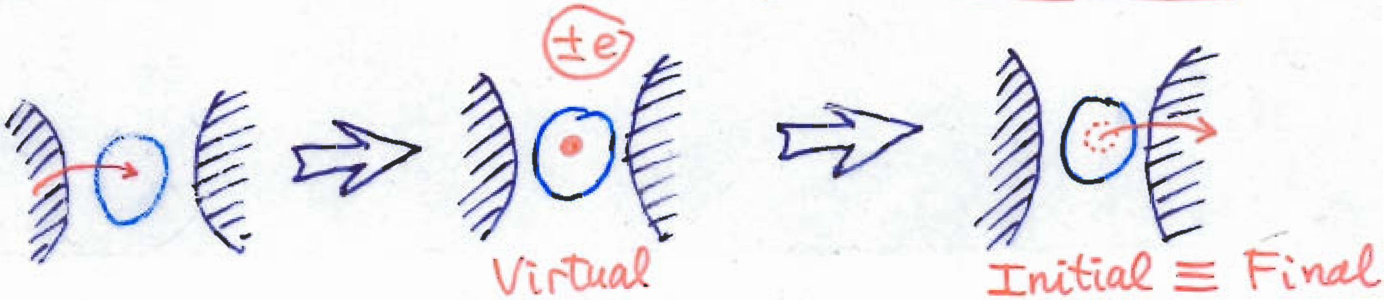
- Sequential hopping: $\boxed{G_{SH} \sim g\, e^{-E_c/T}}$



- Inelastic Cotunneling: $\boxed{G_{IC} \sim g^2 \left(\frac{T}{E_c}\right)^2}$



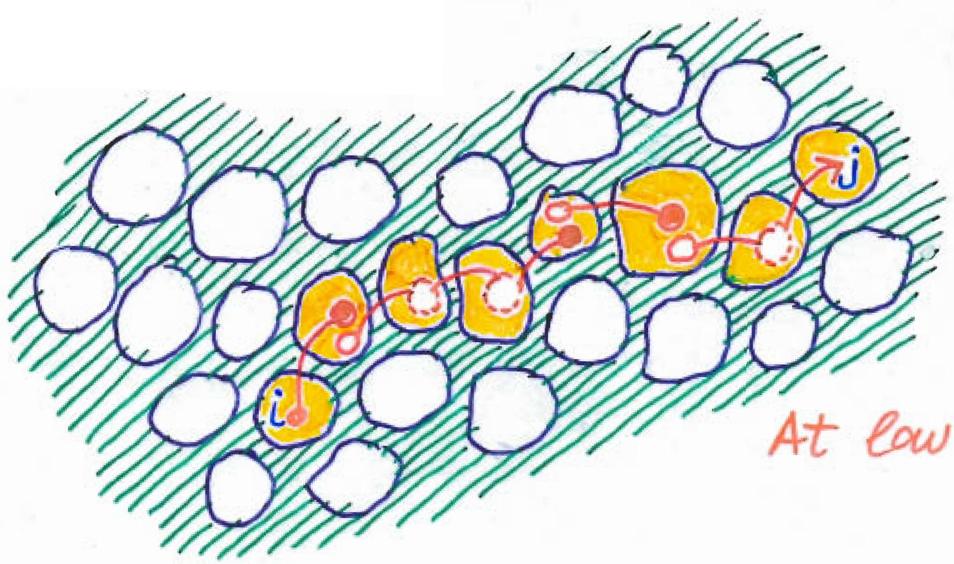
- Elastic cotunneling: $\boxed{G_{EC} \sim g^2 \left(\frac{\delta}{E_c}\right)}$



Elastic - Inelastic Crossover at

$$\boxed{T_{el} \sim \sqrt{E_c \delta}}$$

Distant cotunneling



ΔE_{ij} is small

Chain of $N+2$ intermedidte grains

At low T $N \gg 1$

- Both elastic and inelastic cotunnelings can coexist
- Arbitrary sequence of elementary processes

Effective conductance between distant grains

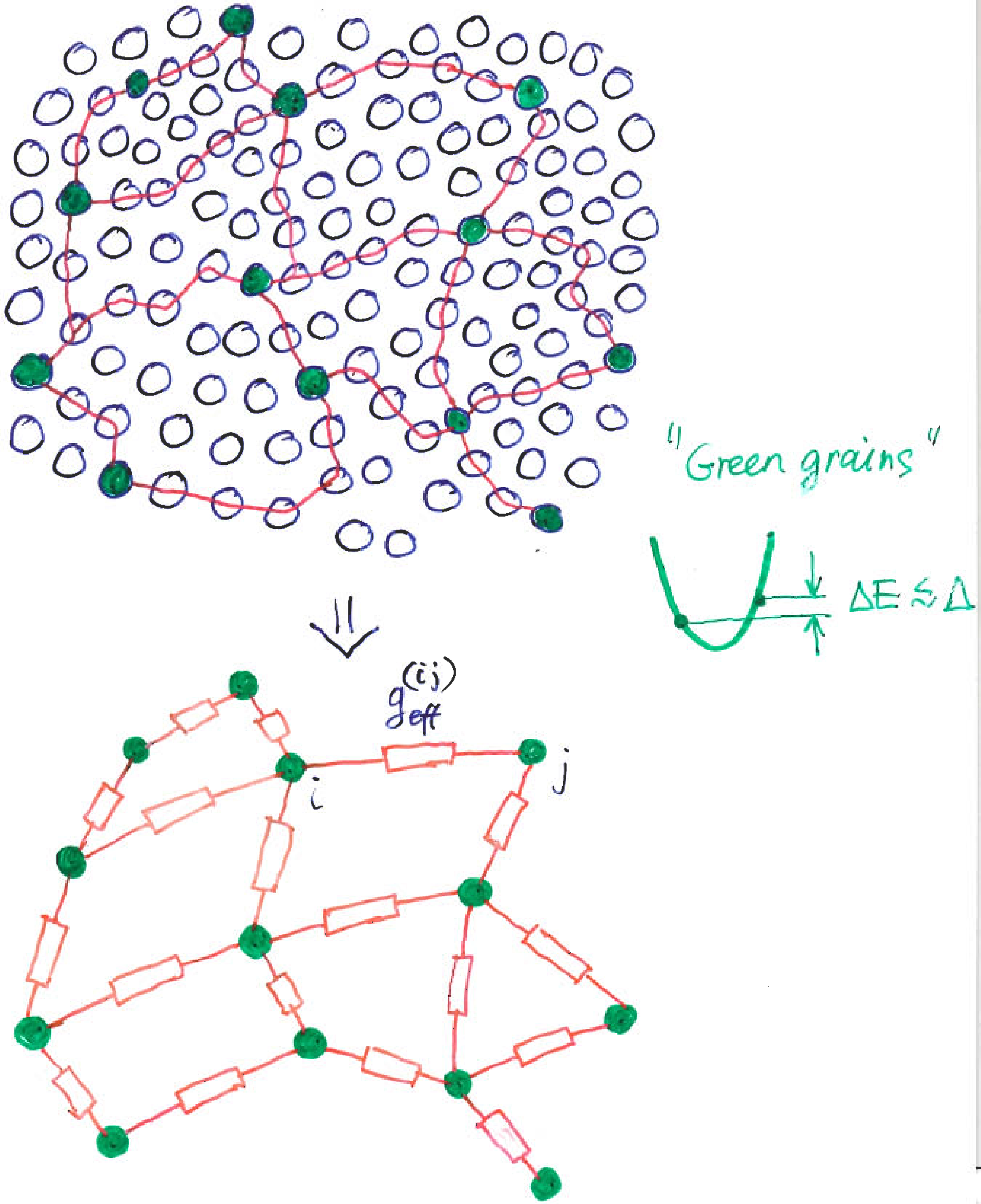
$$g_{ij}^{(eff)} \sim \exp\left\{-\frac{\epsilon_{ij}}{T} - \frac{2 r_{ij}}{a_{eff}(r_{ij}\,\Delta E_{ij})}\right\}$$

$$\frac{a}{a_{eff}} = \begin{cases} \ln\left[\dfrac{8\pi^2}{A_1 g}\,\dfrac{E_c}{\delta}\right] & \text{all elastic} \\ \ln\left[\dfrac{16\pi^2}{e^2 A_2 g}\left(\dfrac{E_c}{\Delta E_{ij}}\,\dfrac{r_{ij}}{a}\right)^2\right] & \text{all inelastic} \end{cases}$$

Random resistors network with $g_{ij}^{(eff)}$

General arguments by Mott + Coulomb gap (Efros & Shklovskii)

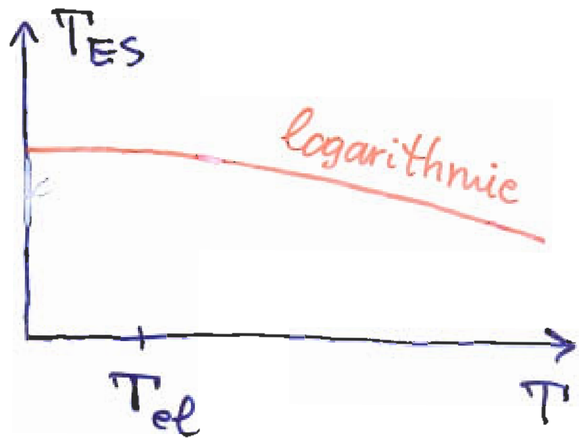
Variable Range Cotunneling subnetwork:



- "Δ-subnetwork": ● - only those grains with charging energies $\epsilon_i < \Delta$
- Find $\sigma(\Delta)$
- Optimize with respect to Δ.

The modified Efros-Shklovskii law

$$\sigma \propto \exp\left\{-\left(\frac{T_{ES}(T)}{T}\right)^{1/2}\right\}$$



$$T_{ES} \sim \frac{e^2}{\epsilon_{eff}\, a_{eff}} \sim E_c\, \mathcal{L}(T)$$

$$\mathcal{L}(T) = \begin{cases} \ln\left[\frac{16\pi^2}{e^2 A_2 g}\left(\frac{E_c}{\mathcal{L}(T)\, T}\right)^2\right] & \text{for } T > T_{el} \\ \ln\left[\frac{8\pi^2}{A_1 g}\,\frac{E_c}{\delta}\right] & \text{for } T < T_{el} \end{cases}$$

$$\boxed{T_{el} \sim \frac{\sqrt{E_c\,\delta}}{\mathcal{L}}}$$

For realistic parameters $\mathcal{L} \sim 10$ so that $T_{el} \sim 1\,K$

Rough estimate: $T \to 2T \Rightarrow T_{ES} \to 0.8\, T_{ES}$

Schematic derivation of $g_{ij}^{(eff)}$ (M. Feigelman, A.I.'05)

The Hamiltonian

$$\hat{H} = \hat{H}_o + \hat{H}_c + \hat{H}_{tun}$$

- $\hat{H}_o = \sum_i \hat{H}_o^{(i)}$ $\qquad \hat{H}_o^{(i)} = \sum_{\alpha_i \sigma} \epsilon_{\alpha_i} a^+_{\alpha_i \sigma} a_{\alpha_i \sigma}$

 (single-particle states within grains)

- $\hat{H}_c = \frac{1}{2} \sum_{ij} E_{ij}^{(c)} (\hat{N}_i - q_i)(\hat{N}_j - q_j)$

 (random q_i)

- $\hat{H}_{tun} = \sum_{\langle ij \rangle} \hat{H}_{tun}^{(ij)}$ $\qquad \hat{H}_{tun}^{(ij)} = \sum_{\alpha_i \alpha_j \sigma} t_{\alpha_i \alpha_j} a^+_{\alpha_i \sigma} a_{\alpha_j \sigma}$

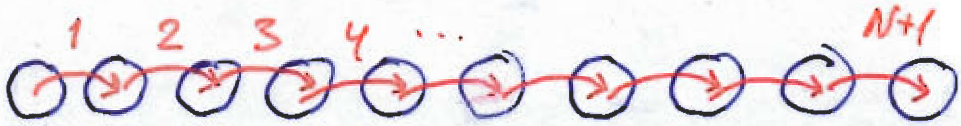
Chaotic grains: irregular fluctuations of $t_{\alpha_i \alpha_j}$:

$$\boxed{\langle t_{\alpha_i \alpha_j} t^*_{\alpha_i' \alpha_j'} \rangle = |t_{ij}|^2 \delta_{\alpha_i \alpha_i'} \delta_{\alpha_j \alpha_j'}}$$

Perturbation theory in $\hat{H}_{tun}$ in $(N+1)^{th}$ order =

= distant cotunneling (at $g \ll 1$)

Composite transition amplitude



N+2 grains

$$A_{ij} \propto \sum_{\{p_k h_k\}} \sum_{\mathcal{T}} \prod_{k=1}^{N+1} t_{h_{k+1} p_k} \prod_{m=1}^{N} \frac{1}{E_m(\{ph\}\mathcal{T})}$$

- $\{p_k h_k\}$ – set of single particle states (1 particle + 1 hole per grain)
- $\mathcal{T}$ – "time sequences" (permutations of $(1,2,3 \cdots N+1)$)
- $E_m(\mathcal{T}\{ph\})$ – energy of m^{th} intermediate state.

<u>In the transition probability $\propto |A_{ij}|^2$:</u>

Strong interference between different $\mathcal{T}$

No interference between different sets $\{ph\}$

$$\boxed{g_{ij}^{(eff)} \propto e^{-\frac{\epsilon_{ij}}{T}} \left(\frac{t}{E_c}\right)^{2N} \sum_{L=0}^{N} \left(\frac{2|\Delta E_{ij}|^2}{\delta^2}\right)^L \left(\frac{E_c}{\delta}\right)^{N-L} \frac{A_1^{N-L} A_2^L C_N^L}{(2L+1)!}} \; *$$

L – number of inelastic processes

A_1, A_2 – constants, depending on distribution of q_i (For strong disorder $A_1 = A_2 = e$)

* is exact for "on site Coulomb" case $E_{ij}^{(c)} = E_i^{(c)} \delta_{ij}$

For general long-range $E_{ij}^{(c)}$ * is qualitatively correct

Conclusions and Outlook

High-resistivity samples

- Inelastic cotunneling dominates the entire experimental T-range (Room – Helium)!
- Modified Efros-Shklovskii law:

  $\boxed{T_{ES}(T) \sim E_c \mathcal{L}(T)}$ $\boxed{\sigma \propto \exp\left[-\left(T_{ES}(T)/T\right)^{1/2}\right]}$
- Inelastic processes destroy interference and suppress magnetoresistance.

Low-resistivity samples

(Still controversial)

- In the "metallic phase" $\left(T > T_{IMT} \sim E_c e^{-\frac{g^2}{4}}\right)$

$$\sigma \sim g\, F\left(\frac{\ln \frac{gE_c}{T}}{g}\right)$$

NO Disorder: $F(x) = 1 - \frac{4x}{z}$ Efetov, Tschersich '03

strong disorder $F(x) = ??$

- "Insulating phase" $(T < T_{IMT})$ –

  First — simple activation

  After that — modified Efros-Shklovskii
- "Ultra-quantum limit" $T < \delta$??

  Not much is understood...