Minigap in superconductor-ferromagnet junctions with inhomogeneous magnetization

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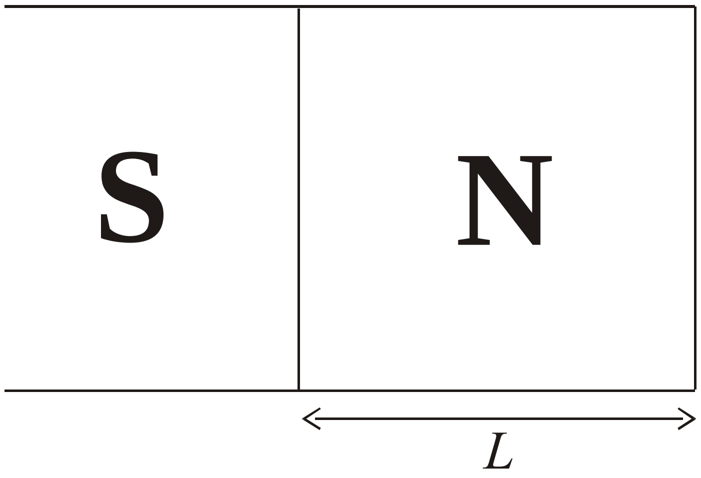
cond-mat/0511299

Trieste Workshop *13 December 2005*

Thouless minigap



Long junction, diffusive limit:

$$L \gg \xi = \sqrt{\frac{D}{\Delta}} \gg l$$

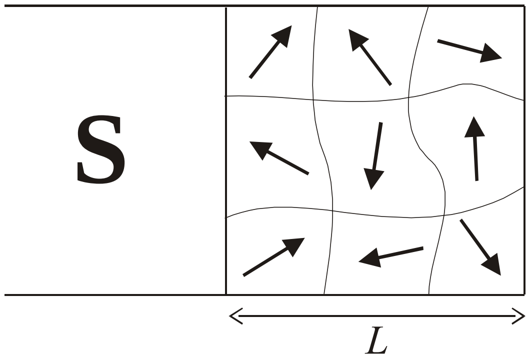
$$E_{\mathrm{Th}} = \frac{D}{L^2}$$ - Thouless energy (inverse diffusion time)

Minigap: $E_g = 0.78\, E_{\mathrm{Th}}$

is linearly closed by the exchange field h in a ferromagnet

Formulation of the problem

1)



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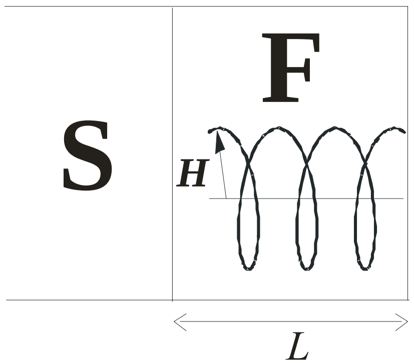

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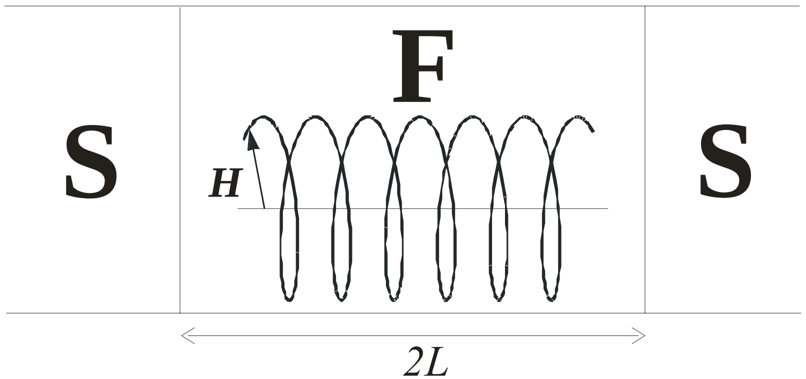


a – domain size

$$E_d = \frac{D}{a^2}$$

We assume

$$a \ll \sqrt{\frac{D}{h}}, \qquad a \ll L$$

or

$$E_d \gg h, \qquad E_d \gg E_{\mathrm{Th}}$$

Usadel equation

$$D\nabla\left(\breve{g}\nabla\breve{g}\right)+iE\left[\hat{\tau}_3\hat{\sigma}_0,\breve{g}\right]-i\left[\hat{\tau}_3(\mathbf{h}\hat{\boldsymbol{\sigma}}),\breve{g}\right]-\Delta\left[\hat{\tau}_1\hat{\sigma}_0,\breve{g}\right]=0$$

$\hat{\tau}$ - Pauli matrices in the Nambu space

$\hat{\sigma}$ - Pauli matrices in the spin space

$\breve{g}$ - 4×4 matrix in the Nambu ⊗ spin space

Without ferromagnetism (at **h** = 0) – only two components:

$$\breve{g} = \hat{\tau}_3\hat{\sigma}_0\, g_0 + \hat{\tau}_1\hat{\sigma}_0\, f_0$$

$\breve{g}^2 = 1$ → $g_0 = \cos\theta, \quad f_0 = \sin\theta$

In the superconductor: $\tan\theta_S = \frac{i\Delta}{E}$

Calculation of the Thouless minigap

In the normal metal: $$\frac{D}{2}\theta'' + iE\sin\theta = 0$$

Boundary conditions: $\theta = \theta_S = \frac{\pi}{2}$ - at the SN interface (at $E << \Delta$)

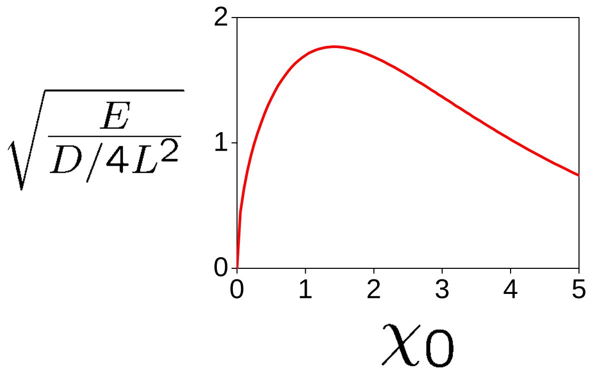
$\theta' = 0$ - at the free sufrace

Below the gap: $\mathrm{Re}\cos\theta = 0$ $\qquad \theta = \frac{\pi}{2} + i\chi$

First integral of the Usadel equation:

$$\frac{D}{4}(\chi')^2 + E\sinh\chi = \text{const} = E\sinh\chi_0$$

$$\sqrt{\frac{E}{D/4L^2}} = \int_0^{\chi_0} \frac{d\chi}{\sqrt{\sinh\chi_0 - \sinh\chi}}$$



SF junction

"Vector" components of the Green function appear:

$$\check{g} = \hat{\tau}_3 \left(g_0 \hat{\sigma}_0 + \mathbf{g} \hat{\boldsymbol{\sigma}} \right) + \hat{\tau}_1 \left(f_0 \hat{\sigma}_0 + \mathbf{f} \hat{\boldsymbol{\sigma}} \right)$$

Generalization of the θ-parameterization:

$$g_0 = M_0 \cos\theta, \qquad \mathbf{g} = i\mathbf{M} \sin\theta$$

$$f_0 = M_0 \sin\theta, \qquad \mathbf{f} = -i\mathbf{M} \cos\theta$$

$$M_0^2 - \mathbf{M}^2 = 1$$

One scalar and one vector Usadel equation:

$$\frac{D}{2} \nabla^2 \theta + M_0 (iE \sin\theta + \Delta \cos\theta) - (\mathbf{hM}) \cos\theta = 0$$

$$\frac{D}{2} (\mathbf{M} \nabla^2 M_0 - M_0 \nabla^2 \mathbf{M}) - \mathbf{M} (iE \cos\theta - \Delta \sin\theta) - \mathbf{h} M_0 \sin\theta = 0$$

Boundary conditions -

SF interface: $\theta = \frac{\pi}{2}, \quad M_0 = 1, \quad \mathbf{M} = 0$

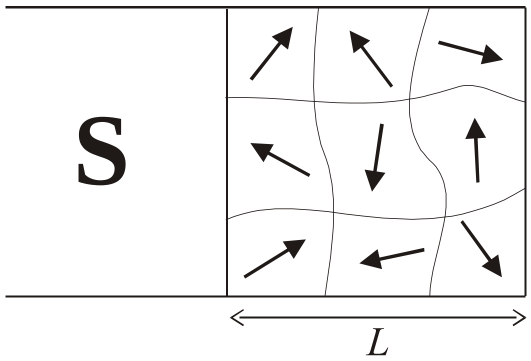
free surface: $\theta' = 0, \quad M_0' = 0, \quad \mathbf{M}' = 0$

Disordered F

Physical idea: effective averaging of **h**, hence small **M**

Linearized vector equation:

$$\left(\frac{D}{2}\nabla^2 + iE\cos\theta\right)\mathbf{M} = -\mathbf{h}\sin\theta$$



Scalar equation:

$$\frac{D}{2}\nabla^2\theta + M_0 iE\sin\theta - (\mathbf{hM})\cos\theta = 0$$

$$\frac{D}{2}\nabla^2\theta + iE\sin\theta - 2\Gamma_{sf}\sin\theta\cos\theta = 0$$

Self-consistency: $\Gamma_{sf} = \left\langle \frac{\mathbf{hM}}{2\sin\theta} \right\rangle$

Validity of linearization: $|\mathbf{M}|^2 \ll \frac{\Gamma_{sf}}{E_{\mathrm{Th}}}$

Neglecting "screening"

$$\left(\frac{D}{2}\nabla^2 + \cancel{iE\cos\theta}\right)\mathbf{M} = -\mathbf{h}\sin\theta$$

When?

Without "screening": $\mathbf{M}(\mathbf{r}) = -\int G(\mathbf{r}-\mathbf{r}_1)\mathbf{h}(\mathbf{r}_1)\sin\theta(\mathbf{r}_1)d^d\mathbf{r}_1$

Error: $\delta\mathbf{M}(\mathbf{r}) = -\int \delta G(\mathbf{r}-\mathbf{r}_1)\mathbf{h}(\mathbf{r}_1)\sin\theta(\mathbf{r}_1)d^d\mathbf{r}_1$

- Green function w/o screening – potential of a point charge: $G_{3D} \propto \frac{1}{r}, \quad G_{2D} \propto \ln r, \quad G_{1D} \propto r$
- δG lives on the scale L
- $\overline{\mathbf{h}(\mathbf{r})\mathbf{h}(\mathbf{r}')}$ decays on a scale $a \ll L$
- $E \lesssim E_{\mathrm{Th}}$ hence $|\theta| \sim 1$

Result: $\frac{\overline{|\delta\mathbf{M}|^2}}{\overline{|\mathbf{M}|^2}} \sim \left(\frac{a}{L}\right)^{d-2}$

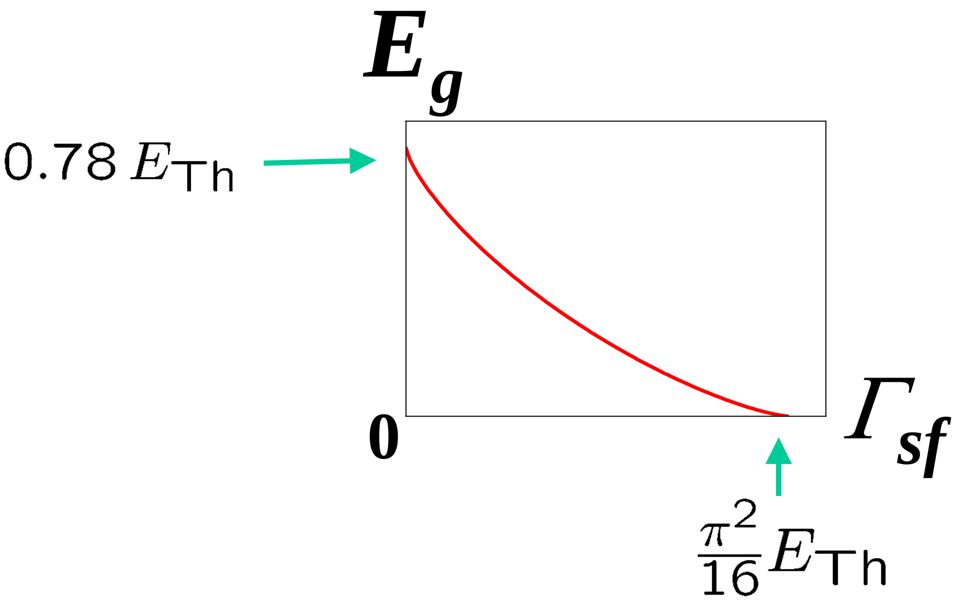
$$\mathbf{M} = -\frac{2}{D}(\nabla^{-2}\mathbf{h})\sin\theta$$

- θ is slow (scale L)
- $\mathbf{h}$ is fast (scale $a \ll L$)

Finally,

$$\Gamma_{sf} = -\frac{\langle \mathbf{h}\nabla^{-2}\mathbf{h}\rangle}{D}$$

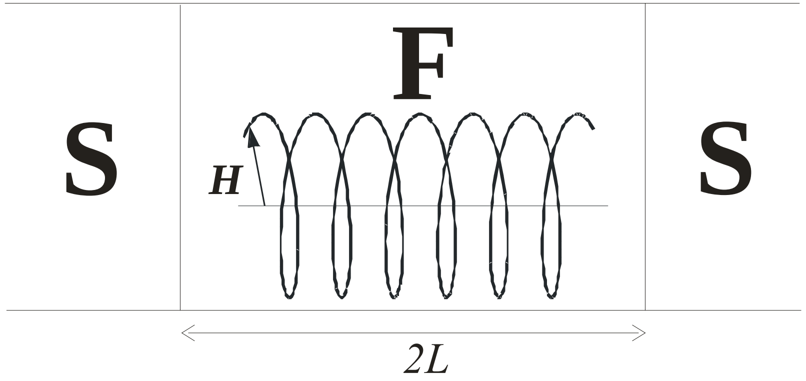
$$\frac{D}{2}\nabla^2\theta + iE\sin\theta - 2\Gamma_{sf}\sin\theta\cos\theta = 0$$



$$\Rightarrow \quad h_c \sim \frac{L}{a}E_{\mathrm{Th}}$$

Spiral 1D order in F

1)



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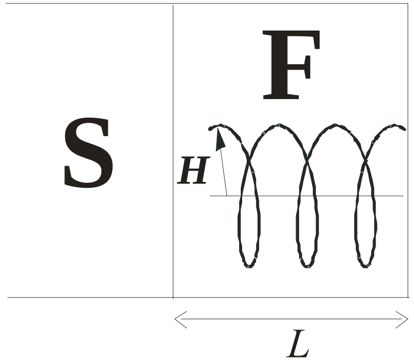


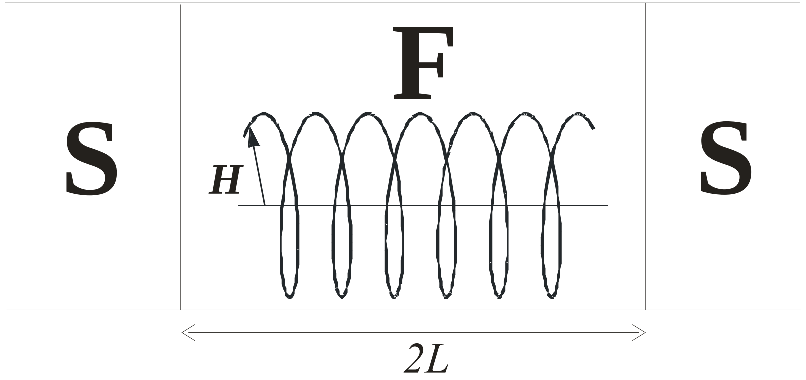
$$\mathbf{h} = h(\cos kz, \sin kz, 0)$$

We cannot neglect "screening" in $\left(\frac{D}{2}\nabla^2 + iE\cos\theta\right)\mathbf{M} = -\mathbf{h}\sin\theta$

At the same time, we can find the gap closing point considering $E = 0$ and looking for branching out of nonzero $\theta - \frac{\pi}{2}$

- SFS – linearization over **M** still works
- SF – full nonlinear problem

Spiral SFS



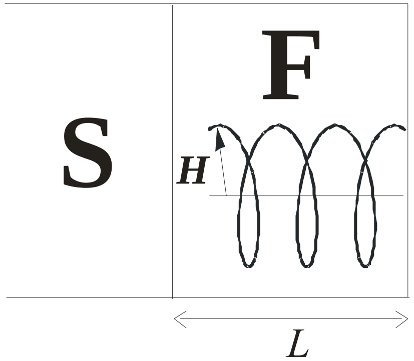
$E = 0$ – no problem with "screening": $$\frac{D}{2}\nabla^2 \mathbf{M} = -\mathbf{h}$$

Oscillating part of the solution: $$\mathbf{M} = \frac{2\mathbf{h}}{Dk^2}$$

Scalar Usadel equation: $$\left[\frac{D}{2}\nabla_z^2 + \frac{2h^2}{Dk^2}\right]\left(\theta - \frac{\pi}{2}\right) = 0, \qquad \theta(0) = \theta(2L) = \frac{\pi}{2}$$

The minigap closes at $$h_c^{\mathrm{SFS}} = \left(\frac{\pi}{4}\right)\frac{Dk}{L}$$

Spiral SF



Linearization over M does not work!

Below the minigap, M is real.

Since $\mathbf{M} = (M_1, M_2, 0)$, we introduce

$$m(z) = M_1 + iM_2$$

Look for the gap closing point, $E = 0$:

$$\frac{D}{2}\left(m'' - m\frac{M_0''}{M_0}\right) + he^{ikz} = 0, \qquad M_0 = \sqrt{1 + mm^*}$$

Separation of rapidly oscillating modes:

$$m = b_0 + b_1 e^{ikz} + b_{-1} e^{-ikz} + \ldots, \qquad \frac{b_0}{b_{\pm 1}} \sim kL \gg 1$$

Eliminate $b_{\pm 1}$, leave only b_0

Effective equation for the smooth component of θ:

$$\left[\nabla_z^2 + \left(\frac{2h}{Dk}\right)^2 \left(1 + |b_0|^2\right)\right] \left(\theta - \frac{\pi}{2}\right) = 0$$

with boundary conditions

$$\theta(0) - \frac{\pi}{2} = 0, \quad \theta'(L) = \frac{2h}{Dk}\left[\theta(L) - \frac{\pi}{2}\right] |b_0|$$

The minigap closes at $$h_c^{SF} = 0.3\frac{Dk}{L}$$

Conclusions

- Minigap in superconductor-ferromagnet junctions with inhomogeneous magnetization survives up to

$$h_c \sim \frac{L}{a} E_{\mathsf{Th}} \gg E_{\mathsf{Th}}$$

In the case of disordered magnetization, the full set of "triplet" equations reduces to the scalar Usadel equation with the spin-flip rate

$$\Gamma_{sf} = -\frac{\left\langle \mathbf{h} \nabla^{-2} \mathbf{h} \right\rangle}{D}$$