Entropy Dynamics in heisenberg chains

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In collaboration with

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Outline

- Entanglement in condensed matter systems

- Entanglement and quantum phase transitions

    * Bipartite entanglement

    * Block entropy

- Statics & Dynamics

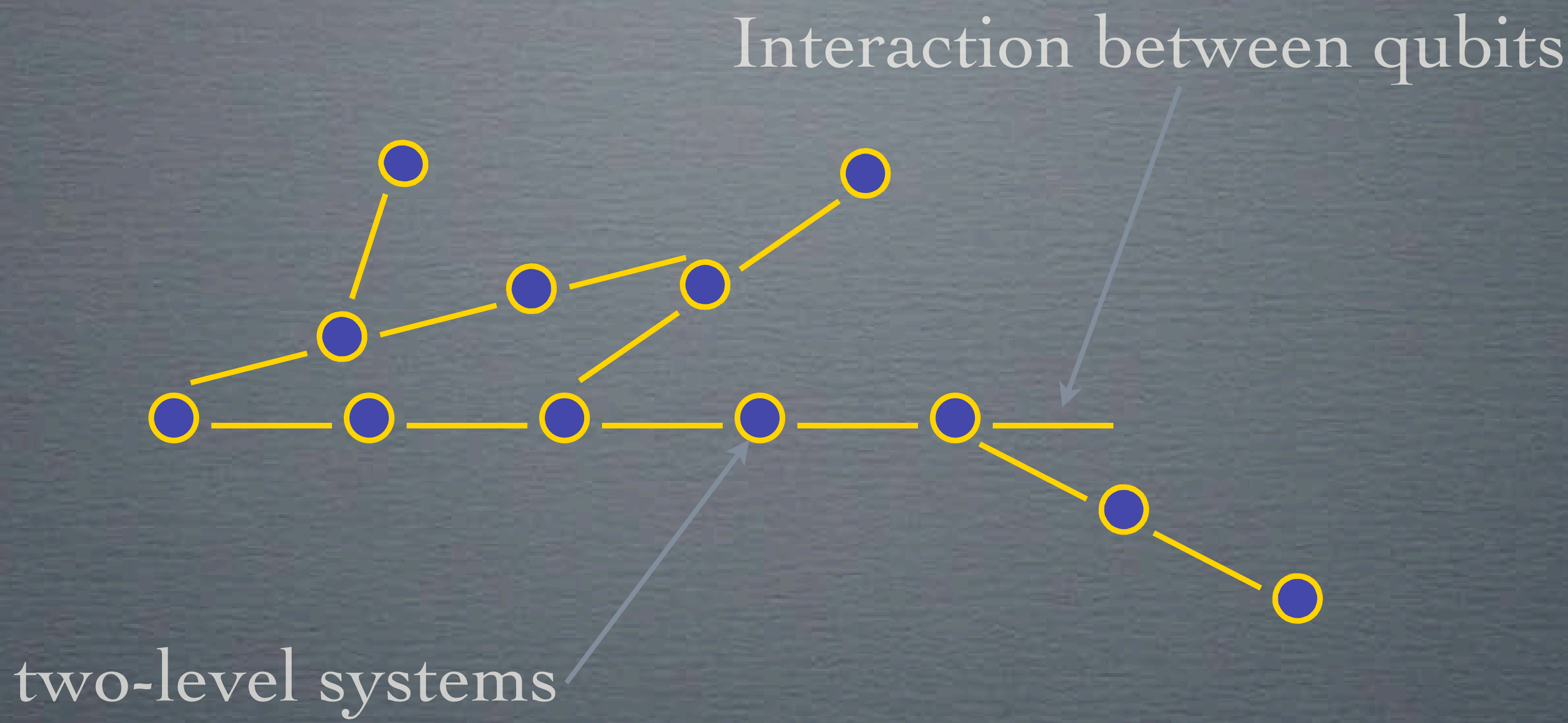
Entanglement in Condensed Matter

- Spin Systems
- Superconductivity
- Quantum Hall Effect
- ....

Characterization of condensed phases

Collective phenomena in Quantum Information and Quantum Communication

Quantum Computer



Quantum Phase Transitions

- T=0 phase transition driven by a coupling constant "g" of the system

Correlations diverge as $|g - g_c|^{-n}$

- The phase transition reflects in a change of the ground state wavefunction -

?? Change in the entanglement properties of the ground state ??

Entanglement and quantum critical systems

A. Osterloh, L. Amico, G. Falci, and R. Fazio '02
T. J. Osborne and M. A. Nielsen '02
G. Vidal, J. I. Latorre, E. Rico, A. Kitaev '02
F. Verstraete, M. A. Martin-Delgado, and J. I. Cirac '03
F. Verstraete, M. Popp, and J. I. Cirac '04
J. Vidal, G. Palacios, and R. Mosseri '04
Y. Chen, P. Zanardi, Z.D. Wang, F.C. Zhang ' 04
P. Calabrese, and J. Cardy 04
V. E. Korepin, 04
R. Somma, G. Ortiz, H. Barnum, E. Knill, and L. Viola, '04
L.-A. Wu, M. S. Sarandy, and D. A. Lidar '04
T. Roscilde, P. Verrucchi, A. Fubini, S. Haas, and V. Tognetti '04
W. Duer, L. Hartmann, M. Hein, M. Lewenstein, and H.-J. Briegel ' 05
J. Eisert, M. Cramer '05
I. Peschel and J. Zhao, '05
A. Anfossi, P. Giorda, A. Montorsi, and F. Traversa '05
O. Guehne and G. Toth '05
T.-C. Wei, D. Das, S. Mukhopadyay, S. Vishveshwara, and P. M. Goldbart '05
...

How to measure entanglement?

- Entanglement between two spins in the network (bipartite)

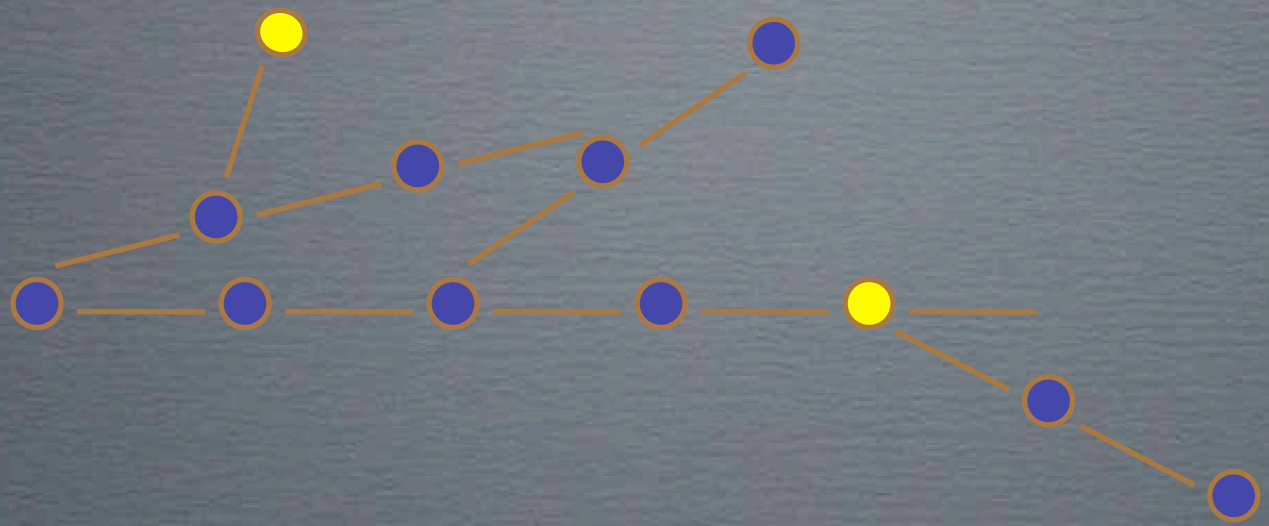
- Multipartite entanglement

- Block entropy

- Localizable entanglement

- ...

Bipartite entanglement



The state of the two selected spins is mixed

Measure of mixed state entanglement for two spin-1/2 states

- Separable state

$$| \alpha\rangle = | 00\rangle$$

- "NOT"

$$| \beta\rangle = | 11\rangle$$

$$\langle\beta | \alpha\rangle = 0$$

-Entangled state state

$$| \alpha\rangle = \frac{1}{\sqrt{2}}(| 00\rangle + | 11\rangle)$$

- "NOT"

$$| \beta\rangle = \frac{1}{\sqrt{2}}(| 00\rangle + | 11\rangle)$$

$$\langle\beta | \alpha\rangle = 1$$

Concurrence between spins at sites i and j

- construct

$$R = \rho(i,j)\tilde{\rho}(i,j)$$

where

$$\tilde{\rho} \doteq \sigma^y \otimes \sigma^y \rho^* \sigma^y \otimes \sigma^y$$

- the concurrence is defined as

$$C(i,j) = \max\{0, \lambda_1(i,j) - \lambda_2(i,j) - \lambda_(i,j) - \lambda_4(i,j)\}$$

where λ s are the eigenvalues of R in ascending order

Ising chain in a transverse field

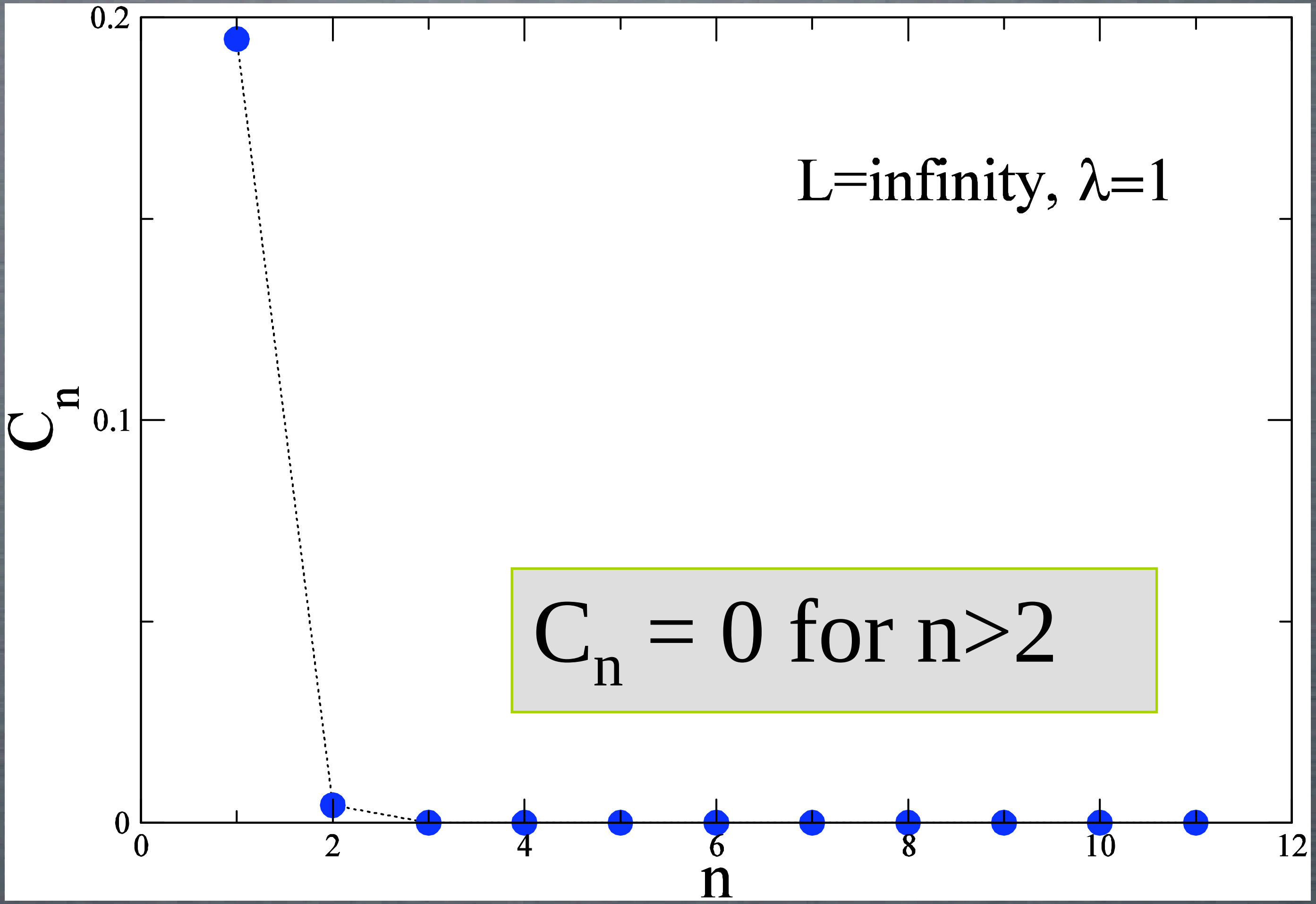
$$H = -\frac{J}{2}\sum_{i=1}^{N}(1-\gamma)\sigma_i^x\sigma_{i+1}^x + (1+\gamma)\sigma_i^y\sigma_{i+1}^y - h\sum_{i=1}^{N}\sigma_i^z$$

$$\lambda = \frac{J}{2h}$$

Exact solution - free fermions

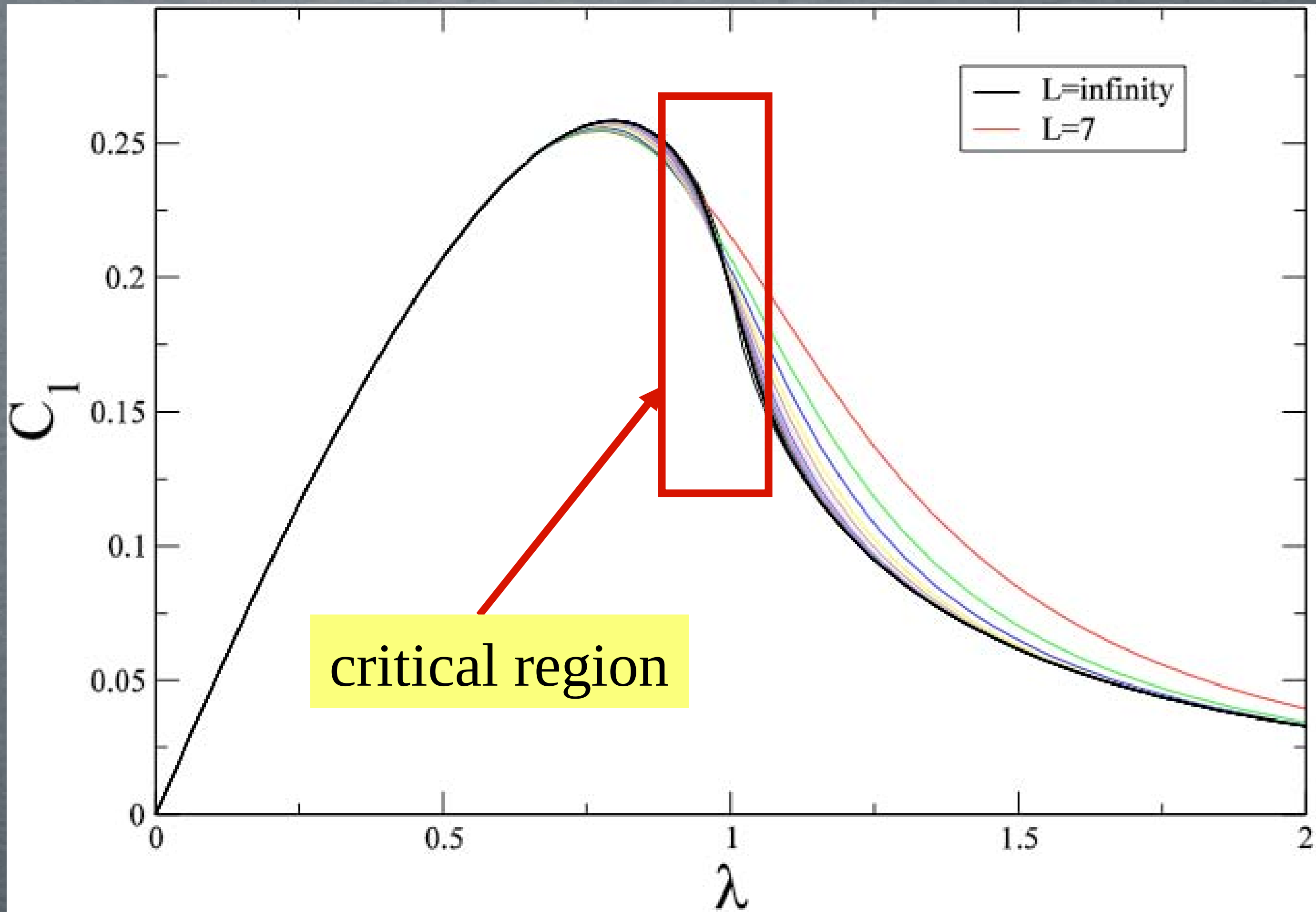
Pfeuty ‘70

Entanglement correlation lenght



A. Osterloh, L. Amico, G. Falci, and R. Fazio '02

Next-neighbour concurrence



A. Osterloh, L. Amico, G. Falci, and R. Fazio '02

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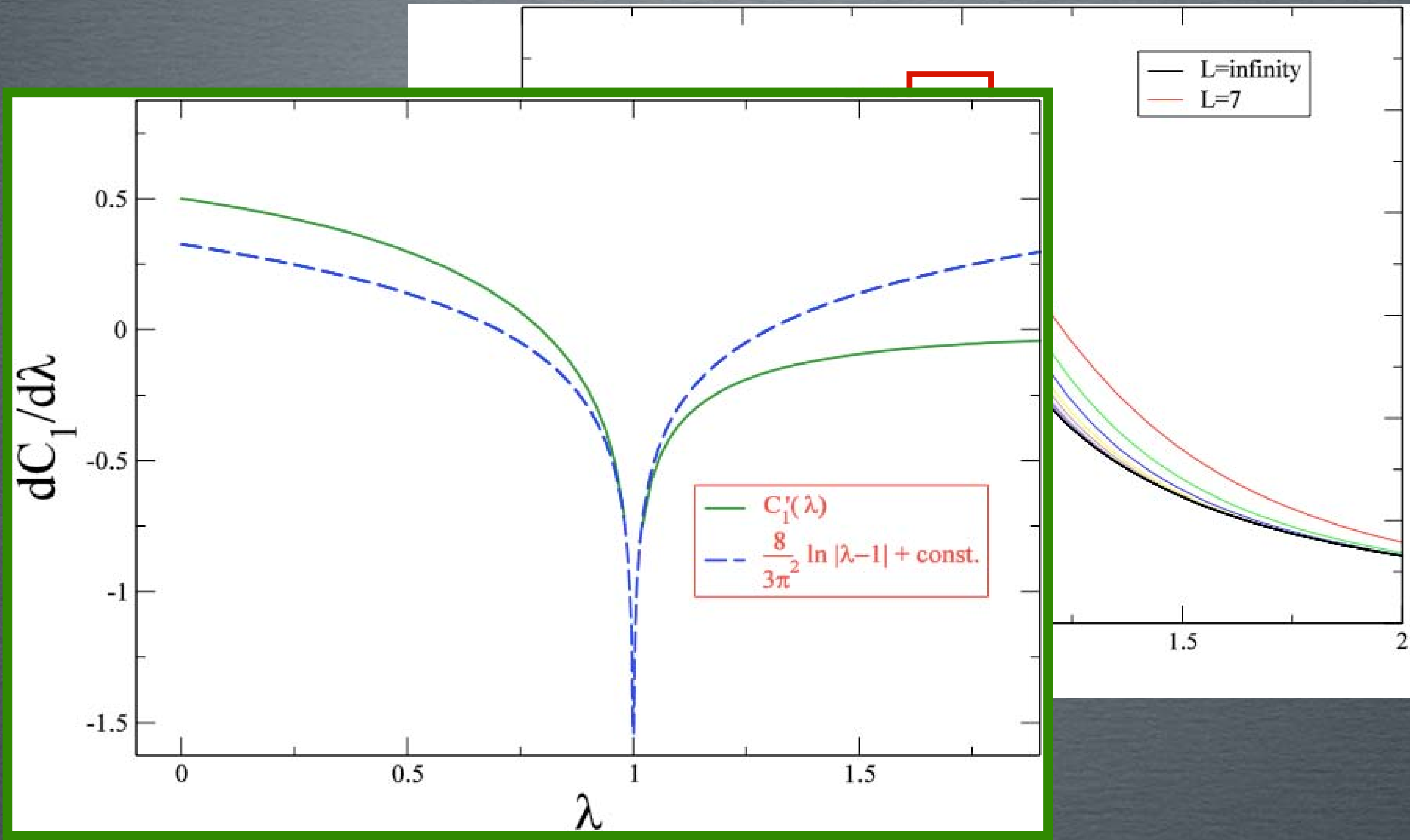
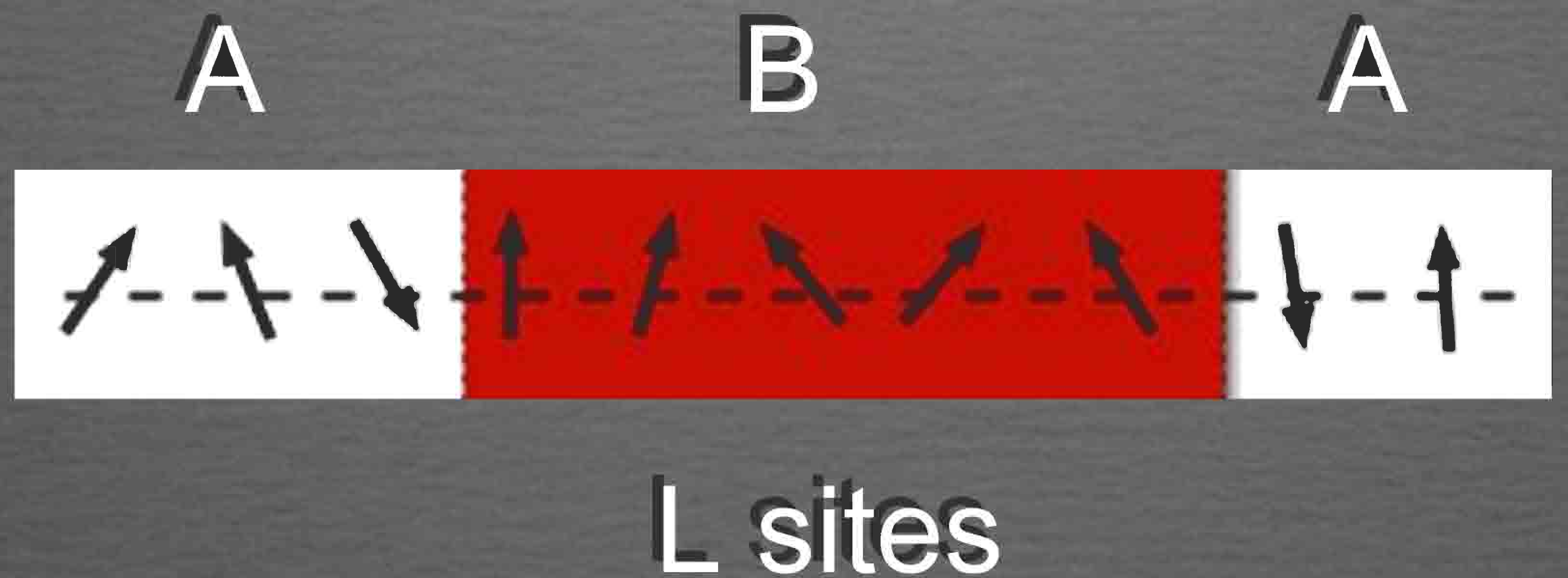


A. Osterloh, L. Amico, G. Falci, and R. Fazio '02

Block entropy

Ground State: $|\Psi_{GS}\rangle$



$$S(\rho_L) \equiv -tr(\rho_L log \rho_L)$$

$$\rho_L = tr_{N-L}(|\Psi_{GS}\rangle\langle\Psi_{GS}|)$$

Block entropy

$$S(\rho_L) \equiv -tr(\rho_L log \rho_L)$$

$$S = \frac{c}{6} \log_2 \left[\frac{N}{\pi} \sin\left(\frac{\pi}{N} L \right) \right] + A$$

- C. Holzhey, F. Larsen, and F. Wilczek ‘94
- G. Vidal, J. I. Latorre, E. Rico, A. Kitaev ‘02
- V. E. Korepin ‘04
- P. Calabrese, and J. Cardy ‘04
- I. Peschel and J. Zhao ‘05

Heisenberg Model

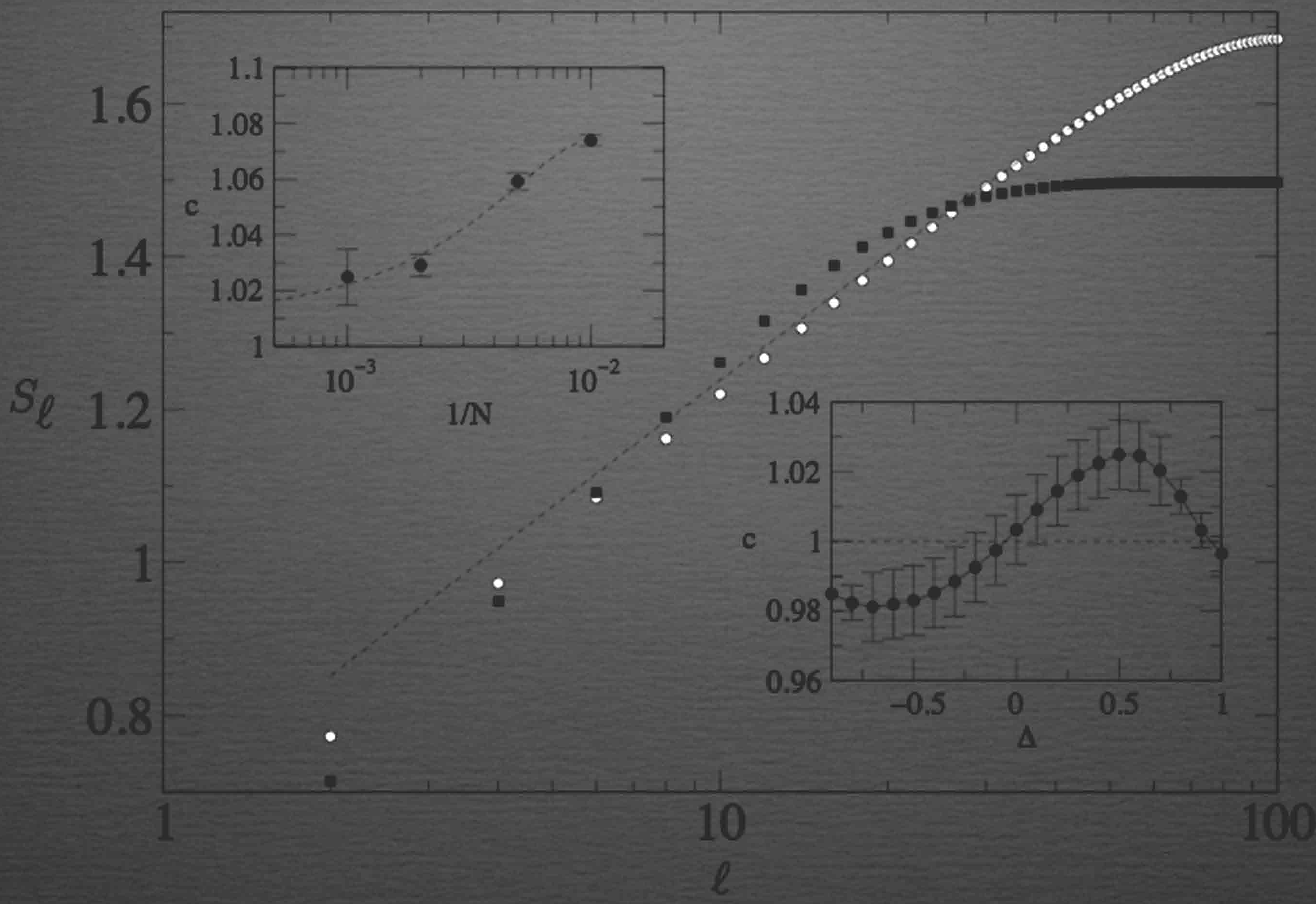
Our results:
- numerical calculations based on DMRG

$$H = \sum_i J_i \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta\, \sigma_i^z \sigma_{i+1}^z \right)$$

$J_i = J$ Constant couplings

$J_i \in [0, J]$ Random HM

Entropy scaling - clean case



Entropy scaling - disordered case

The entropy should scale as in the clean case with an "effective" central charge

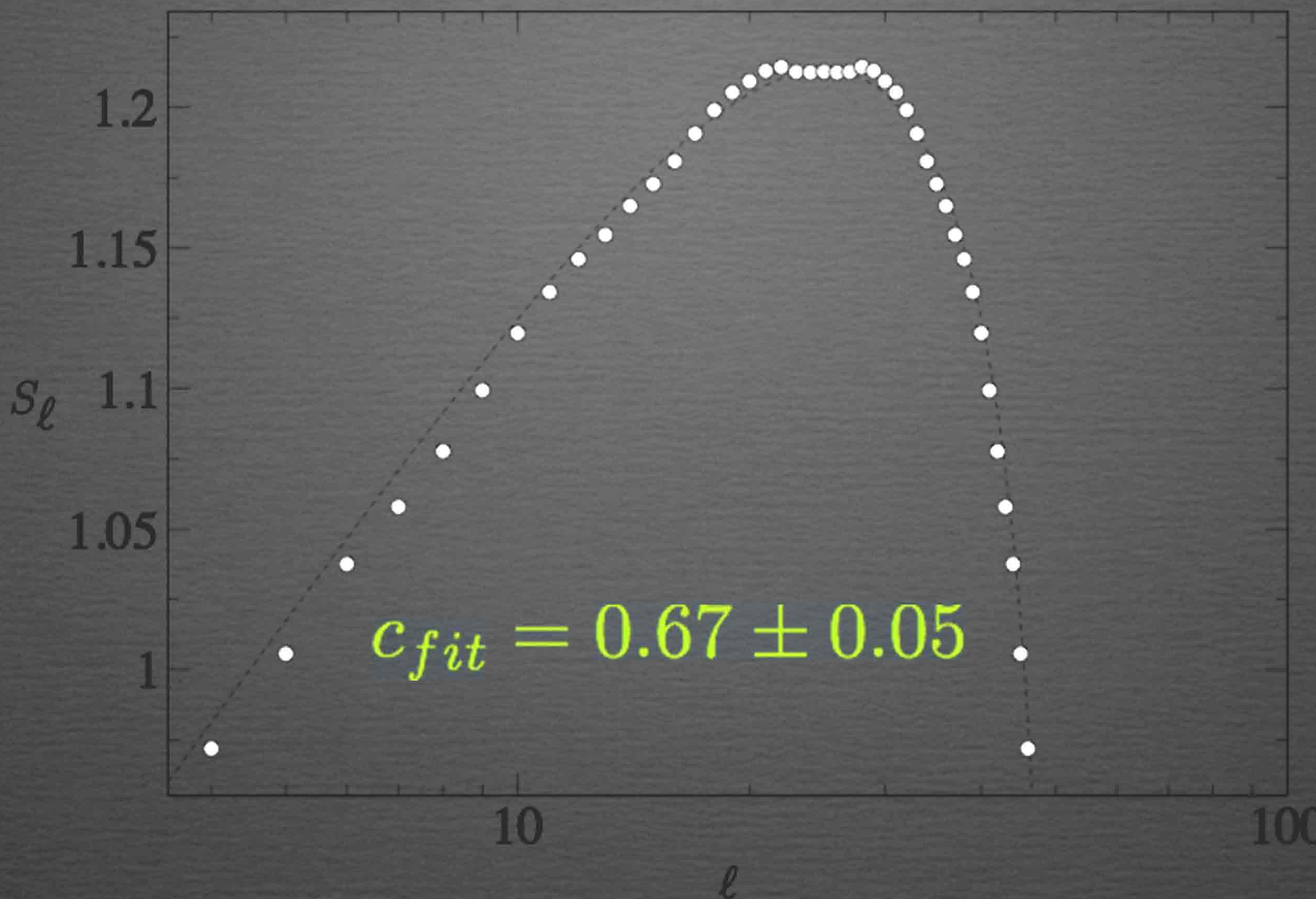
$$c_{eff} = c \ln 2$$

G. Refael and J. Moore '04

Tested numerically for the XX model

N. Laflorencie '05

Entropy scaling - disordered case



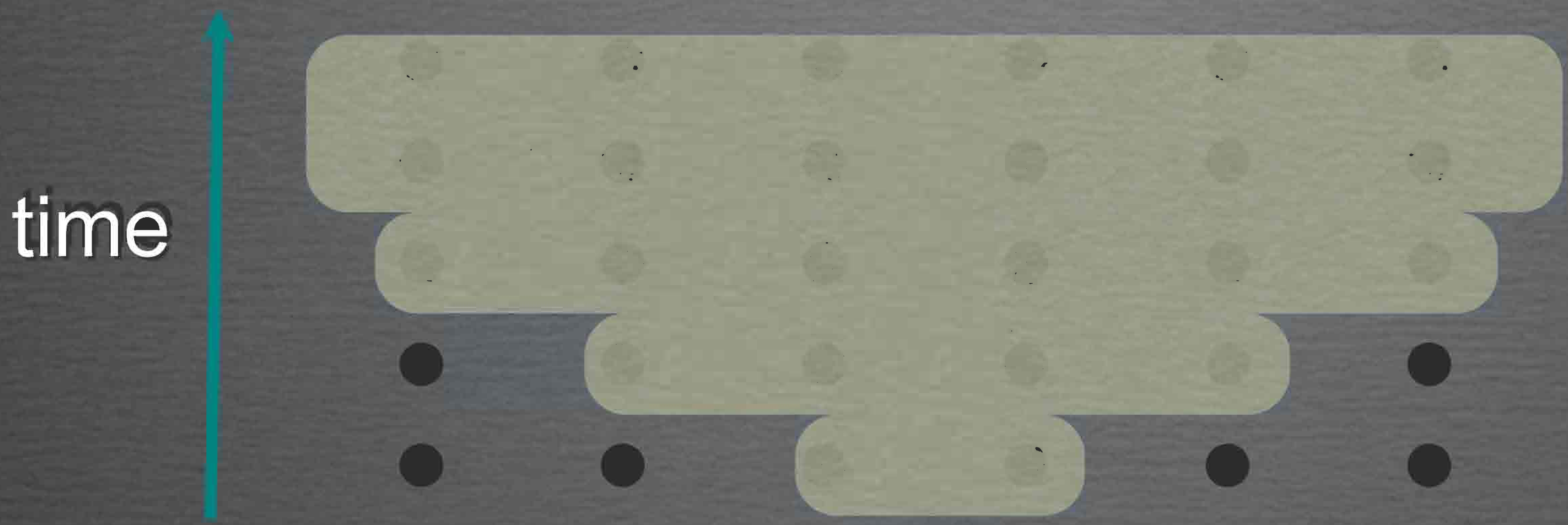
$N = 50 \quad 10^4$ realizations

Dynamics of Entanglement

* Prepare the system in a state (not an eigenstate) with all the entanglement localized in a given part of the chain

* Sudden quench of some of the couplings of the model Hamiltonian.

Dynamics of Entanglement



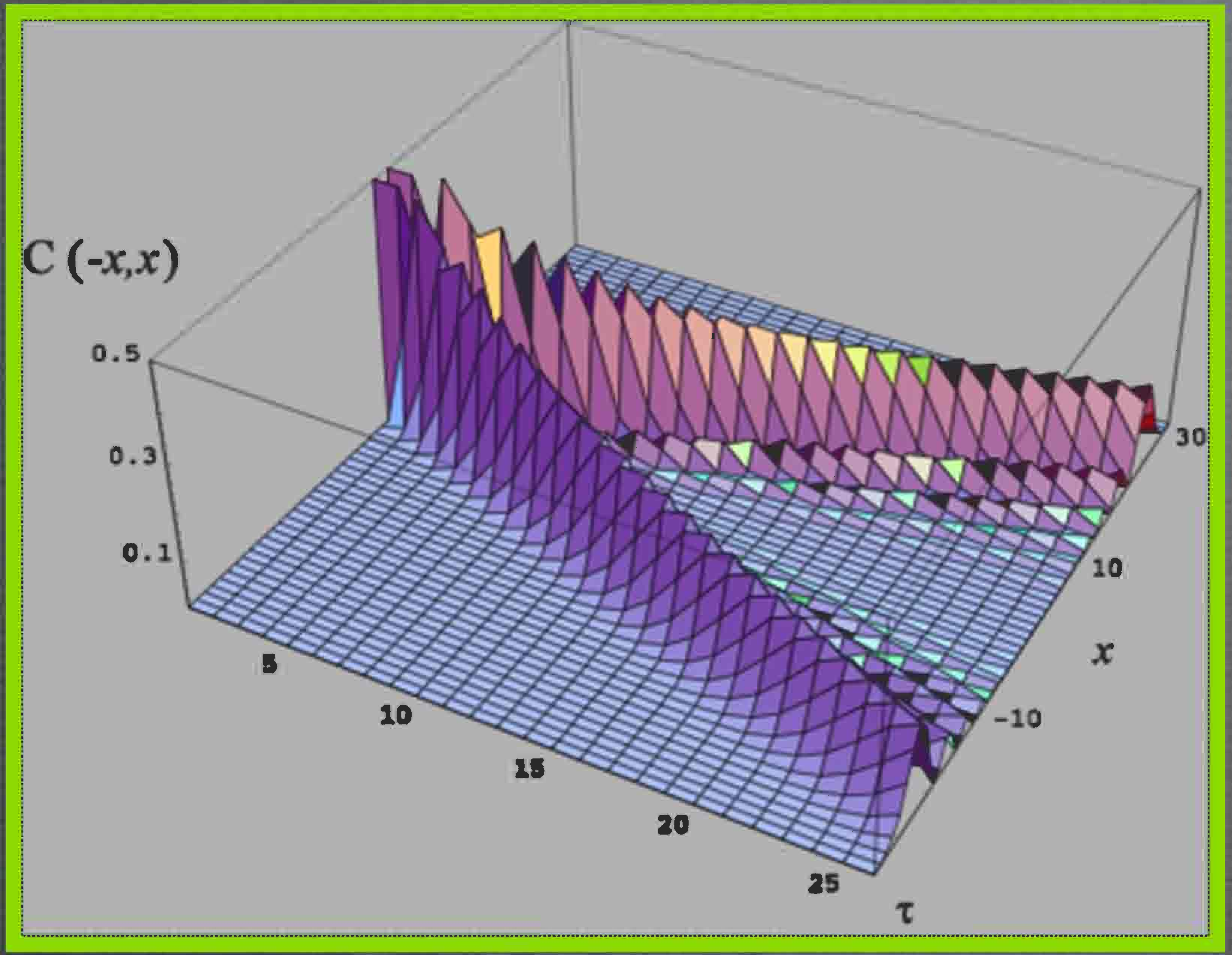
$$| \beta\rangle = \frac{1}{\sqrt{2}}(| 00\rangle + | 11\rangle)$$

Prepare the system in a state (not an eigenstate) with all the entanglement localized in a given part of the chain

L. Amico, A. Osterloh, F. Plastina, R. Fazio and G. M. Palma '04

Dynamics of Entanglement

$$\Delta = 0$$



Prepare the system in a state (not an eigenstate) with all the entanglement localized in a given part of the chain

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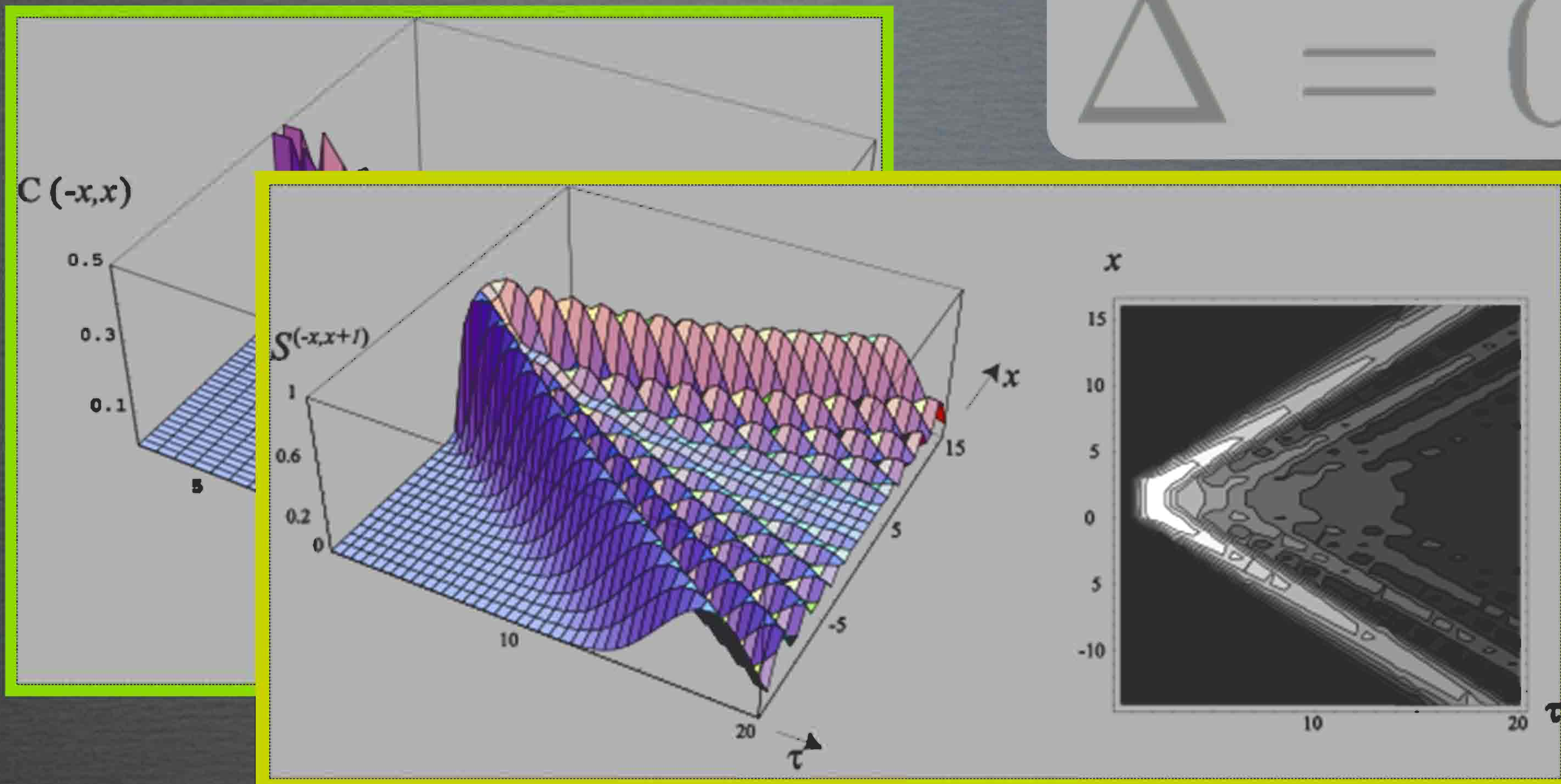
Dynamics of Entanglement

$\Delta = 0$



Prepare the system in a state (not an eigenstate) with all the entanglement localized in a given part of the chain

L. Amico, A. Osterloh, F. Plastina, R. Fazio and G. M. Palma '04

Block entropy - SUMMARY

Static

critical scaling

$$S_L \sim \frac{c}{3} log_2 L$$

Dynamic

Entropy increase is proportional to quench

Entropy saturates at t^*

t^* depends on L and velocity

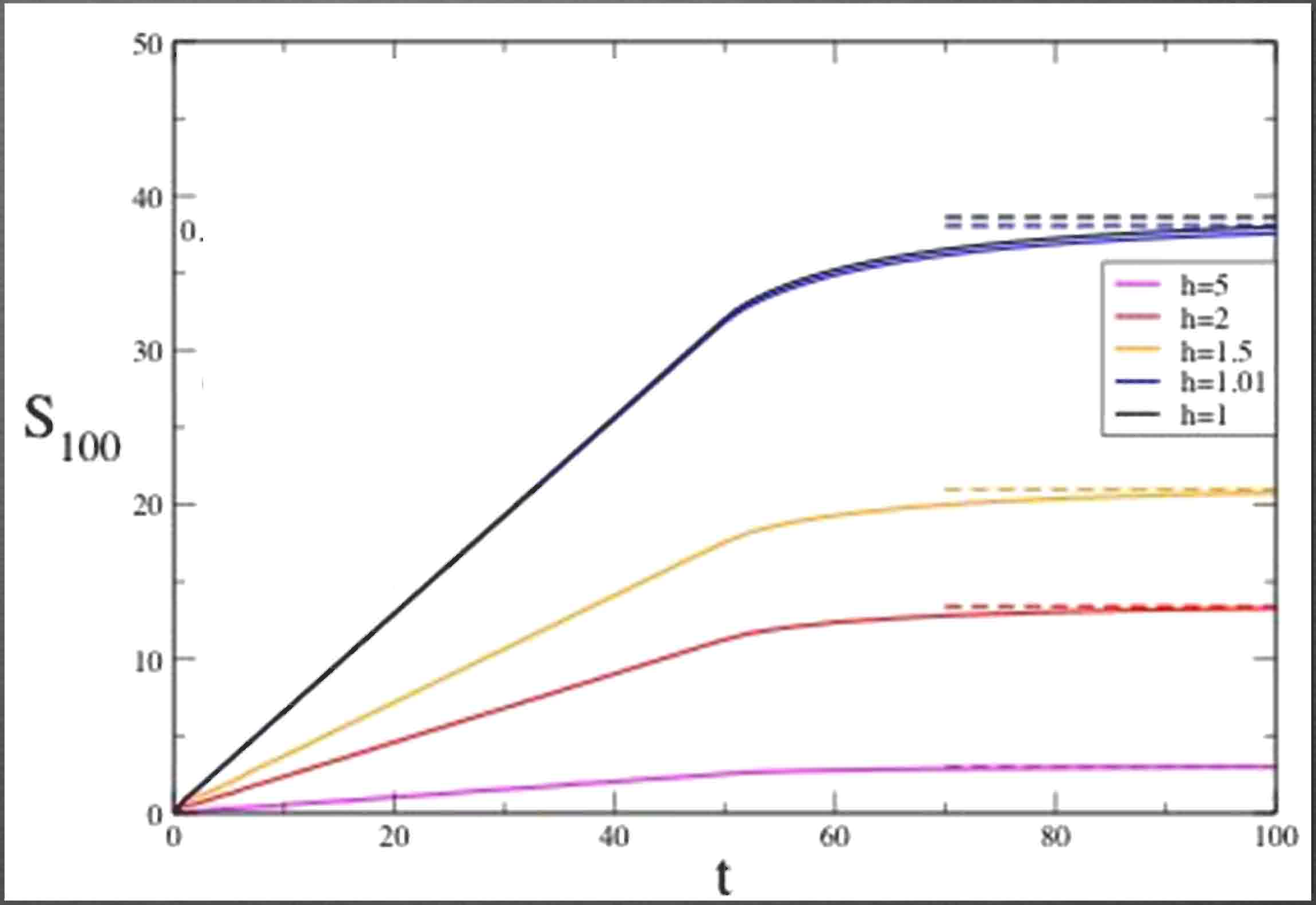
Dynamics of block entropy

Instantaneous Quench

$\lambda_0 = \infty \rightarrow \lambda_1$

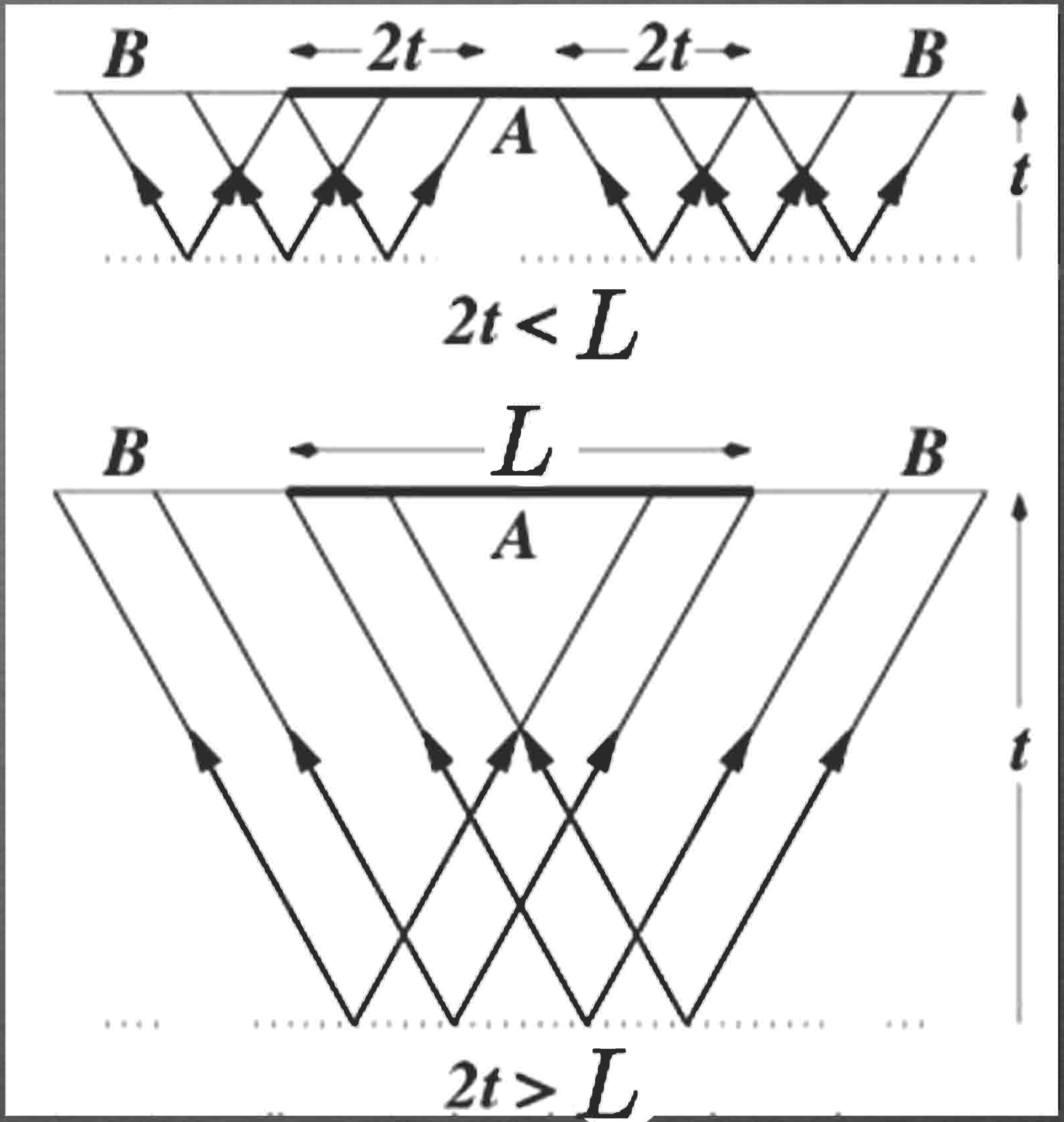
t=0 GS: separable state

$vt^* = L/2$



P. Calabrese, J. Cardy '05

Physical picture

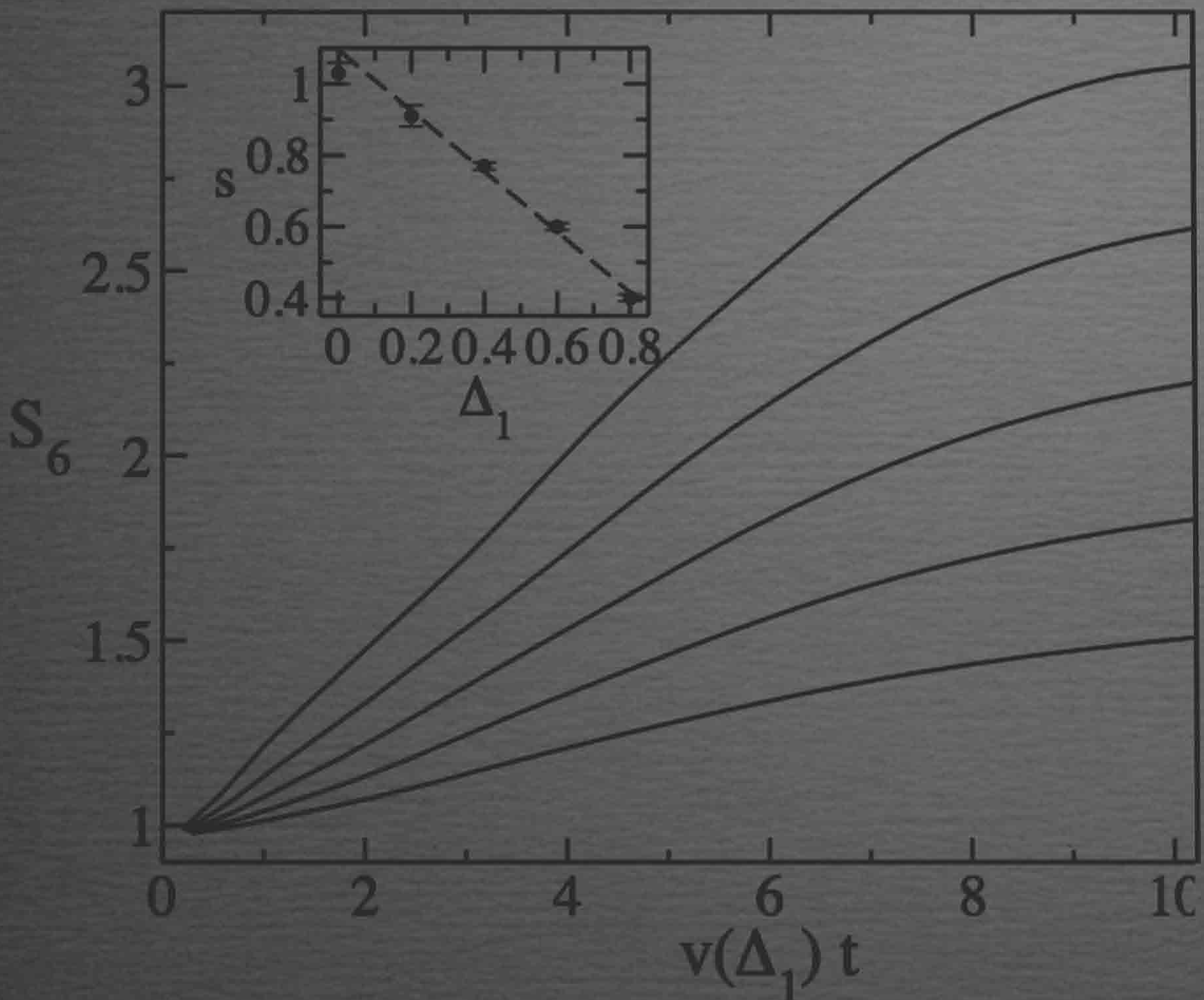


$$S_L \propto t \qquad t < t^*$$

$$S_L \propto L \qquad t > t^*$$

P. Calabrese, J. Cardy ‘ 05

Dynamics of block entropy - DMRG results



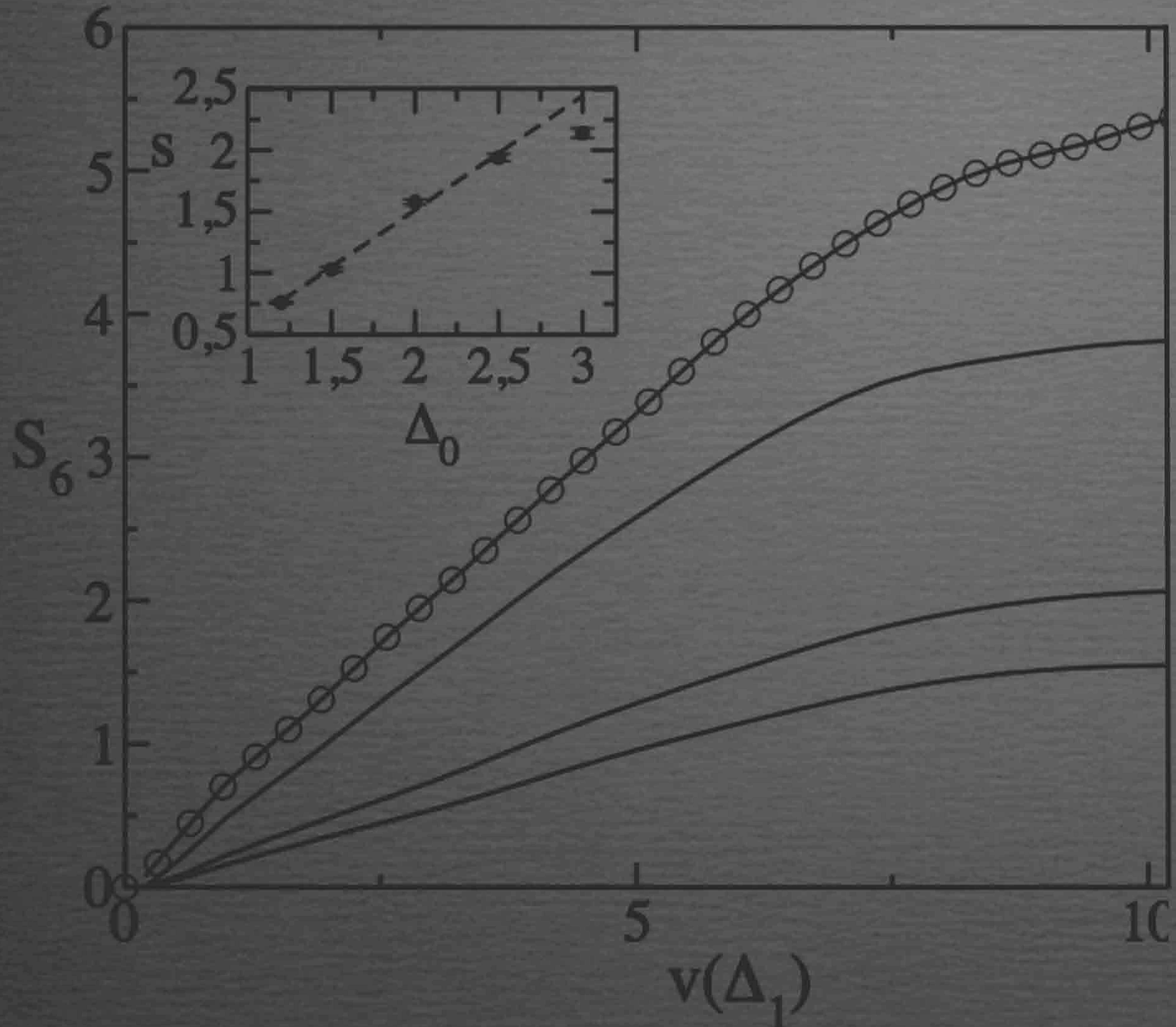
In this case - saturation at vt* = L

$$v(\Delta) = 2J\pi\frac{sin\theta}{\theta}$$

$$cos\theta = \Delta_1$$

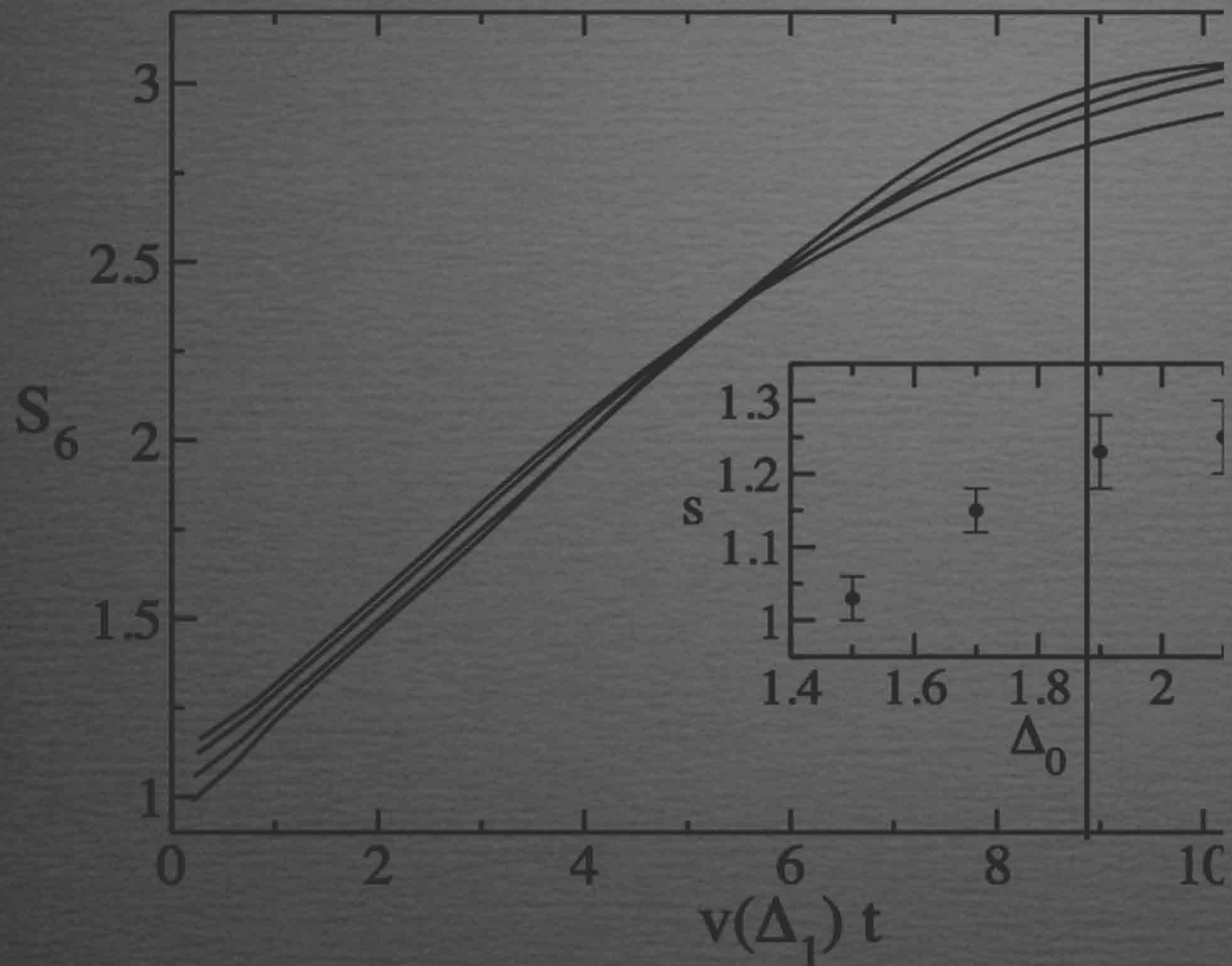
$$N = 50 \quad \Delta_0 = 1.5 \quad \Delta_1 = 0.0, 0.2, 0.4, 0.6, 0.8$$

Dynamics of block entropy - DMRG results



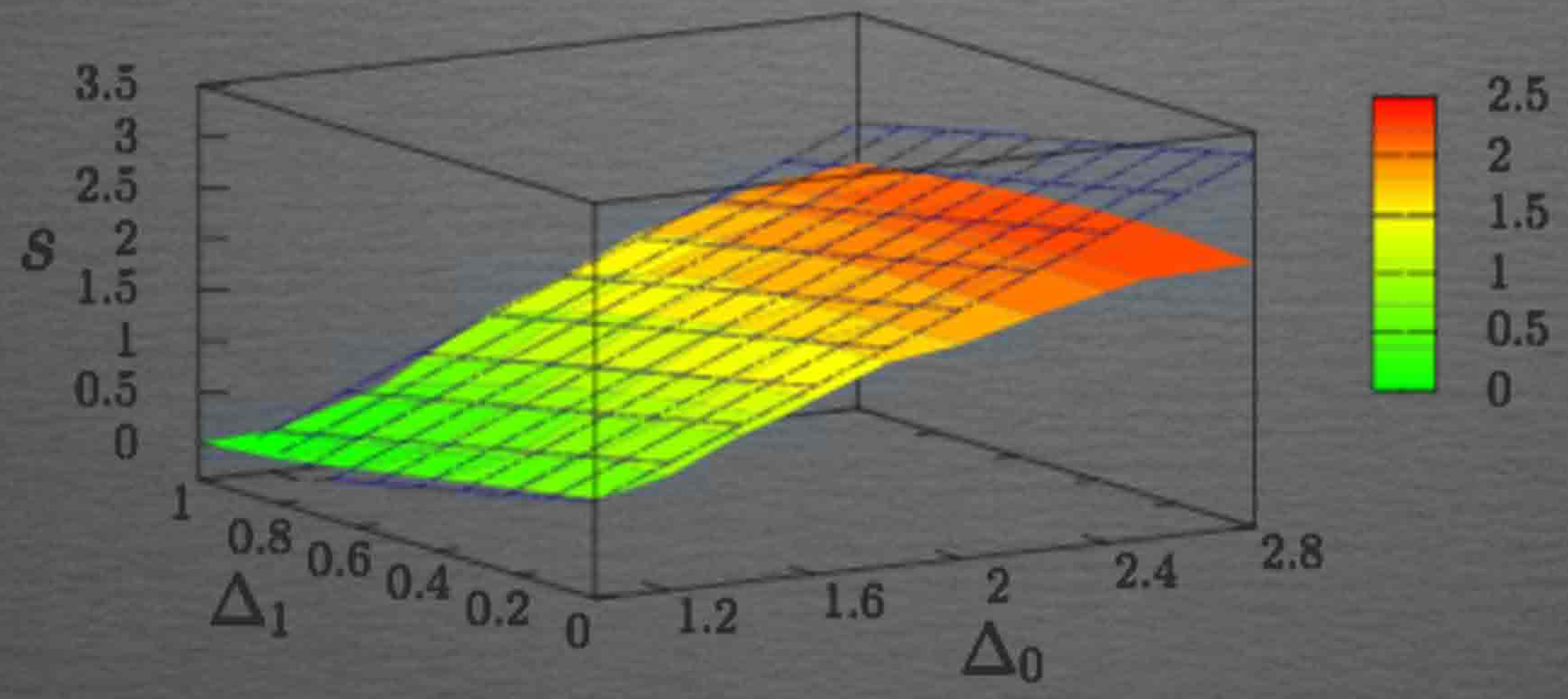
$$\Delta_1 = 0.0 \quad \Delta_0 = 1.2, 1.5, 3.0, \infty$$

Dynamics of block entropy - DMRG results



$$\Delta_0 - \Delta_1 = 1.5 \quad \Delta_0 = 1.5, 1.7, 1.9, 2.1$$

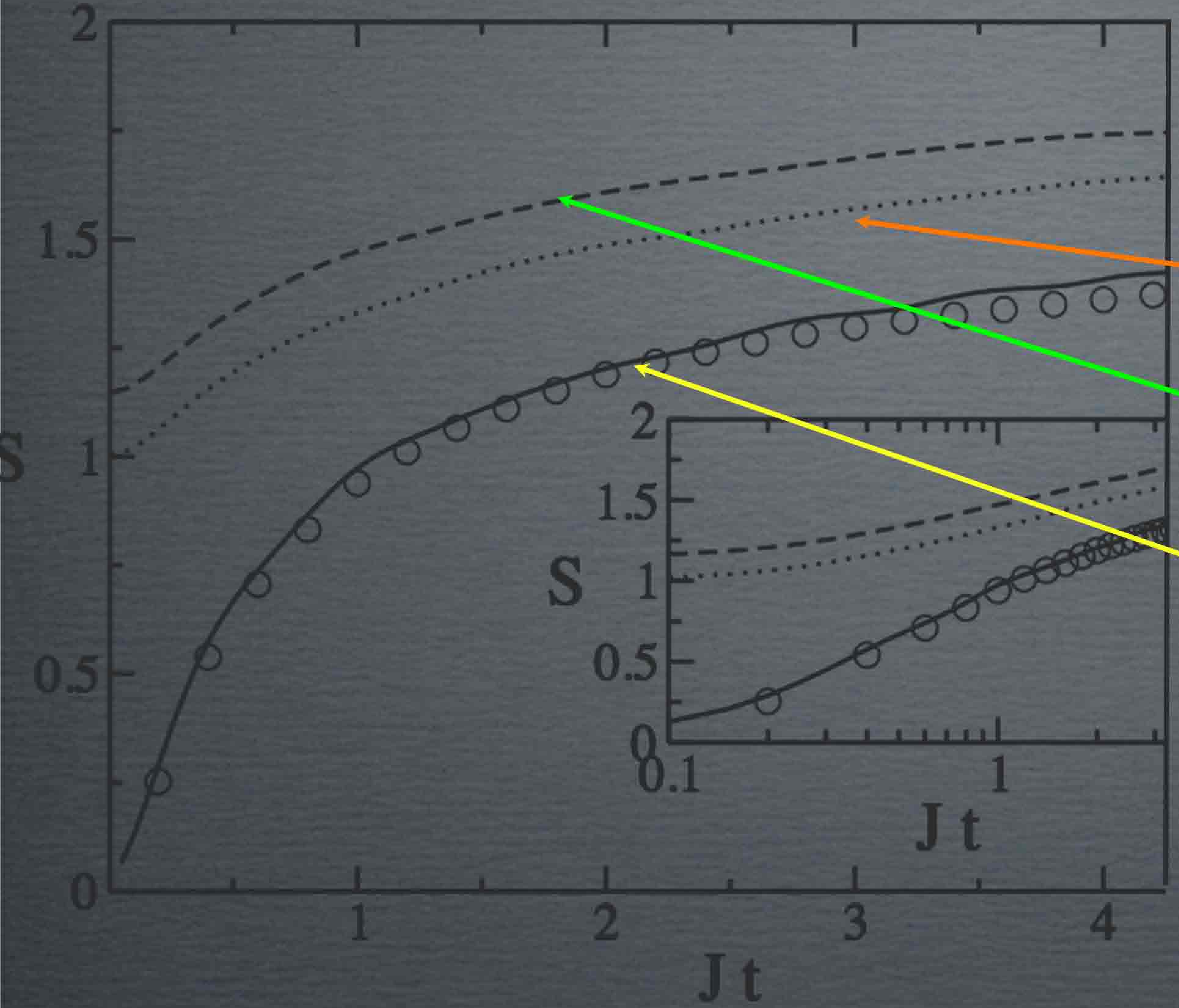
Dynamics of block entropy - DMRG results



The slope of the linear increase of the entropy

$$s = 1.50\Delta_0 - 0.84\Delta_1 - 0.90$$

Dynamics of block entropy - DMRG results



Disordered chains

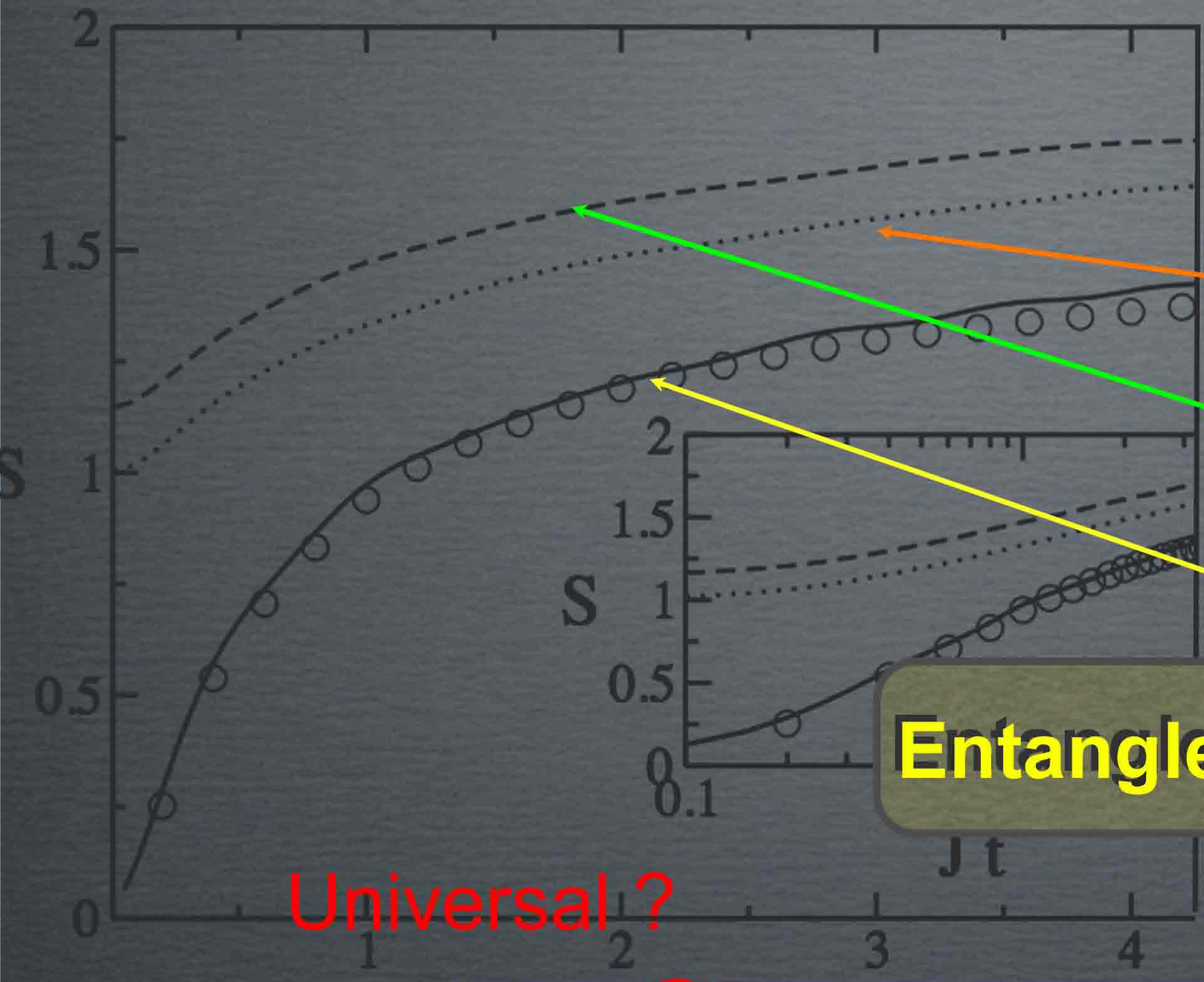
$\Delta_0 = 2 \quad \Delta_1 = 0 \quad L = 10$

$\Delta_0 = 2 \quad \Delta_1 = 0 \quad L = 20$

$\Delta_0 = +\infty \quad \Delta_1 = 0 \quad L = 10$

Dynamics of block entropy - DMRG results

Disordered chains



$\Delta_0 = 2 \quad \Delta_1 = 0 \quad L = 10$

$\Delta_0 = 2 \quad \Delta_1 = 0 \quad L = 20$

$\Delta_0 = +\infty \quad \Delta_1 = 0 \quad L = 10$

Entanglement localization ?

Universal ?

$$S = \frac{1}{2} \ln[-\frac{1}{Jt} + b(L)] + a$$

$\sim 1/L^2$

SUMMARY

- Entanglement very sensitive to quantum critical points

- Numerical analysis of the block entropy for the Heisenberg model

  * In the clean - linear increase with time

  * In the disordered case - entanglement localization?