Bosonization and renormalization group for a clean Fermi gas with a repulsion in arbitrary dimensions.

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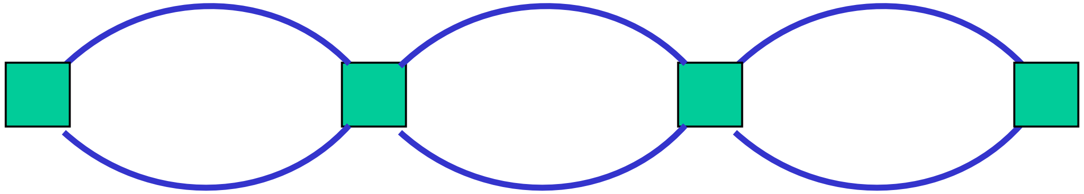
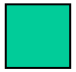
Contents

1. New method of bosonization in $d>1$ including spin excitations.
2. New logarithmic contributions due to spin excitations, renormalization group.
3. Non-Fermi liquid behavior of a Fermi liquid. Temperature dependent spin relaxation.

The main assumption of the Fermi liquid theory:



 = Constant (weakly depends on temperature)

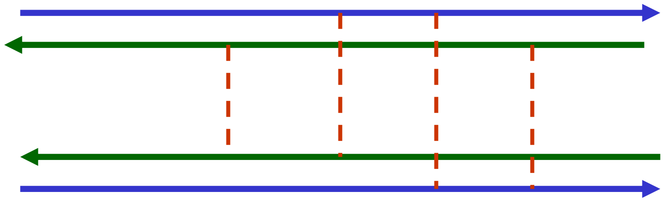
Not always true: e.g. for an attraction one has a Cooper instability.

What about repulsion?

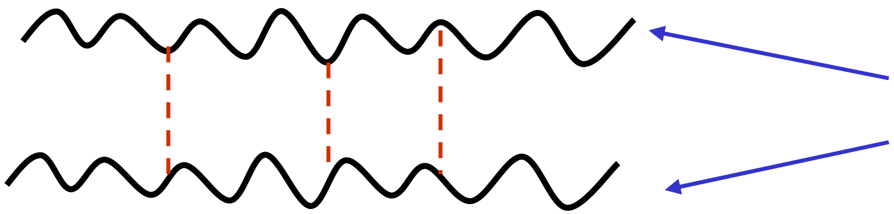
Nothing in the weak coupling limit (common knowledge).

In this work: there are non-trivial correlations for repulsion in the weak coupling limit and even an instability!

Everything comes not from two electron correlations but from four electron ones! ⇨ A reason why nobody (?) has noticed these instabilities before.



Equivalent representation



Spin excitations

Reduction from electrons to collective excitations: Bosonization

The scheme of the method.

1. Singling out slow varying $k < k_c \le p_F$ pairs in the interaction.

$$\mathcal{L}_{int} \to \tilde{\mathcal{L}}_{int} = \frac{1}{2} \sum_{\sigma,\sigma'} \int dP_1 dP_2 dK \qquad (2.10)$$
$$\{V_2 \chi^*_\sigma (P_1) \chi_\sigma (P_1 + K) \chi^*_{\sigma'} (P_2) \chi_{\sigma'} (P_2 - K)$$
$$- V_1 (\mathbf{p}_1 - \mathbf{p}_2) \chi^*_\sigma (P_1) \chi_{\sigma'} (P_1 + K) \chi^*_{\sigma'} (P_2) \chi_\sigma (P_2 - K)\}$$

No pairs of the type $\chi\chi$, $\chi^*\chi^*$ but they are not necessary because only $V(2p_F)$ and $V(0)$ play a role.

Example of a Cooper logarithm with these vertices.

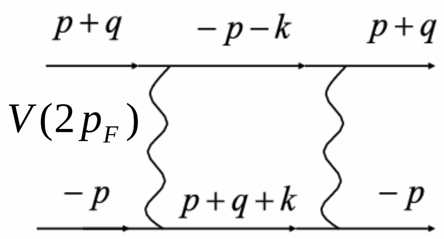


FIG. 2: Superconducting loop. Logarithm comes from small momenta $k < k_c$.

2. Decoupling of the slow pairs by the Hubbard-Stratonovich transformation ⟹ electron motion in a slow field Φ ⟹ writing equations for quasiclassical Green functions

Φ is a 2x2 spin matrix.

$$\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\tau'}-iv_F\mathbf{n}\nabla\right)g_{\mathbf{n}}^{\Phi}\left(\mathbf{R},\tau,\tau'\right) \qquad (2.37)$$
$$+\left(g_{\mathbf{n}}^{\Phi}\left(\mathbf{R};\tau,\tau'\right)\Phi_{\mathbf{n}}\left(\mathbf{R},\tau'\right)-\Phi_{\mathbf{n}}\left(\mathbf{R},\tau\right)g_{\mathbf{n}}^{\Phi}\left(\mathbf{R};\tau,\tau'\right)\right)=0$$

$$g_{\mathbf{n}}^{\Phi}\left(\mathbf{R},\tau,\tau'\right)=i\int_{-\infty}^{\infty}G_{\mathbf{p}}^{\Phi}\left(\mathbf{R},\tau,\tau'\right)\frac{d\xi}{\pi}$$

The solution

$$g_{\mathbf{n}}^{\Phi}\left(\mathbf{R},\tau,\tau'\right)=T_{\mathbf{n}}\left(\mathbf{R},\tau\right)g_0\left(\tau-\tau'\right)T_{\mathbf{n}}^{-1}\left(\mathbf{R},\tau'\right)$$

⟺ Generalization of the Schwinger Ansatz

Equations for

$$M_{\mathbf{n}}\left(x\right)=\frac{\partial T_{\mathbf{n}}\left(x\right)}{\partial\tau}T_{\mathbf{n}}^{-1}\left(x\right)$$

Another representation:

$$M_{\mathbf{n}}\left(x\right)=\rho_{\mathbf{n}}\left(x\right)+\mathbf{S}_{\mathbf{n}}\left(x\right)\sigma$$

$S_n(r,\tau)$ Spin

$\rho_n(r,\tau)$ Charge

Equations for the charge and spin excitations: starting point for the calculations.

$$\left(-\frac{\partial}{\partial\tau}+iv_F\mathbf{n}\nabla_{\mathbf{R}}\right)\rho_{\mathbf{n}}(x)=-i\frac{\partial\varphi_{\mathbf{n}}(x)}{\partial\tau}$$

↔ No interaction leading to infrared divergences.

$$\left(-\frac{\partial}{\partial\tau}+iv_F\mathbf{n}\nabla_{\mathbf{R}}\right)\mathbf{S}_{\mathbf{n}}(x)+2i\left[\mathbf{h}_{\mathbf{n}}(x)\times\mathbf{S}_{\mathbf{n}}(x)\right]=-\frac{\partial\mathbf{h}_{\mathbf{n}}(x)}{\partial\tau}$$

Effective interaction leading to divergent contributions at T=0 (logarithmic in any dimensions)

$$\int\frac{d^dk}{(2\pi)^d}\frac{1}{-i\omega+v_F\vec{k}\vec{n}}\frac{1}{i\omega+v_F\vec{k}\vec{n}}\propto\int\frac{d^{d-1}k_\perp}{(2\pi)^{d-1}}\ln\frac{\varepsilon_F}{T}$$

⇓

Non-trivial effective model for spin excitations.

Logarithmic in any dimensions ⇒ RG treatment

Effective field theory for the spin excitations. (Due to necessity of averaging over **h,** use of supervectors).

The operator $\hat{L}_u = -\frac{\partial}{\partial \tau} + i v_F \left(\mathbf{n} \nabla_{\mathbf{r}} \right) + 2iu\hat{h}$ is not hermitian ⇒ doubling the size of the supervectors ψ

Effective Lagrangian

$$Z = \int \exp\left(-\mathrm{L}\left[\psi\right]\right) D\psi$$

Z -Partition function, L-Lagrangian

$$\mathrm{L}\left[\psi\right] = \mathrm{L}_0\left[\psi\right] + \mathrm{L}_2\left[\psi\right] + \mathrm{L}_3\left[\psi\right] + \mathrm{L}_4\left[\psi\right]$$

$$\mathrm{L}_0\left[\psi\right] = -2i\nu \int \overline{\psi_\alpha\left(X\right)} H_0 \psi_\alpha\left(X\right) dX$$

$$H_0 = \begin{pmatrix} -iv_F\left(\mathbf{m}\nabla\right)\tau_3\Sigma_3 & -\partial/\partial\tau \\ -\partial/\partial\tau & -iv_F\left(\mathbf{m}\nabla\right)\tau_3\Sigma_3 \end{pmatrix}$$

$L_0[\psi]$ -Lagrangian of free excitations

Interaction terms

$$\mathrm{L}_4[\psi] = -8\nu\varepsilon_{\alpha\beta\gamma}\varepsilon_{\alpha\beta_1\gamma_1}\int\left(\overline{\psi_\beta(X)}\tau_3\psi_\gamma(X)\,u\right) \times \tilde{\hat{\Gamma}}\left(u\overline{\psi_{\beta_1}(X)}\tau_3\psi_{\gamma_1}(X)\right)dX \qquad (4$$

$$\varepsilon_{\alpha\beta\gamma} = -\varepsilon_{\alpha\gamma\beta} = -\varepsilon_{\beta\alpha\gamma} = 1$$

$$\mathrm{L}_3[\psi] = -8\nu\sqrt{2}i\varepsilon_{\alpha\beta\gamma}\int\left(\overline{\psi_\beta(X)}\tau_3\psi_\gamma(X)\,u\right) \times \tilde{\hat{\Gamma}}\left(\bar{F}_0\overline{\partial_x(u)}\tau_3\psi_\gamma(X)\right)dX$$

$$\mathrm{L}_2[\psi] = 4i\nu\int\left(\overline{\psi_\alpha(X)}\tau_3\overleftarrow{\partial}_x(u)\,F_0\right) \times \tilde{\hat{\Gamma}}\left(\bar{F}_0\overline{\partial_x(u)}\tau_3\psi_\gamma(X)\right)dX$$

$L_0,\ L_4$ are supersymmetric

$L_2,\ L_3$ violate the supersymmetry ⇒ contribution to thermodynamics

Next step: renormalization group treatment.

Results of the solutions of the RG equations.

Quartic interaction: $\gamma_1(\xi)$ -forward scattering, $\gamma_3(\xi)$ backward scattering

$$\gamma_3(\xi) = \frac{1}{\xi_b^* + \xi},$$

$$\gamma_1(\xi) = \frac{1}{\xi_f^* - \xi}$$

ξ -Logarithmic variable

Cubic interaction:

$$\beta_3^+(\xi) = \frac{1}{\xi_b^* + \xi}, \quad \beta_3^-(\xi) = \frac{\xi_b^*}{(\xi_b^* + \xi)^2}$$

$$\beta_1^+(\xi) = \frac{1}{\xi_f^* - \xi}, \quad \beta_1^-(\xi) = \frac{\xi_f^*}{\left(\xi_f^* - \xi\right)^2}$$

Quadratic interaction:

$\Delta_1(\xi)$ =const= $\Delta(0)$

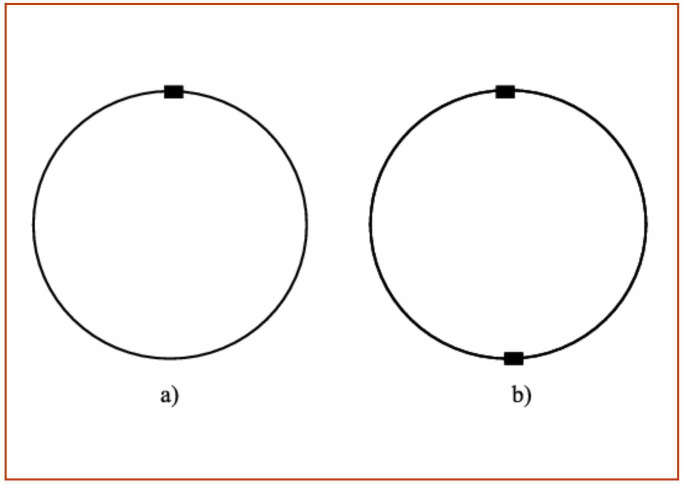
$$\Delta_3^{-+}(\xi) = \frac{2\xi_b^{*2}}{(\xi_b^* + \xi)^3} - \frac{\xi_b^*}{(\xi_b^* + \xi)^2}$$

$$\Delta_3^{+-}(\xi) = \frac{2}{\xi_b^* + \xi} - \frac{1}{\xi_b^*},$$

$$\Delta_3^{++}(\xi) = \Delta_3^{--}(\xi) = \frac{\xi_b^*}{(\xi_b^* + \xi)^2}$$

Results for the specific heat δC

Only the backscattering amplitudes $\Delta_3(\xi)$ contribute to thermodynamics in the one loop approximation.



Thermodynamic potential in the first and second order in the effective Δ

A non-trivial contribution comes from b) only.

$$\delta c_{d=2} = -\frac{3\zeta(3)}{\pi}\left(\frac{T}{\varepsilon_F}\right)^2 \left(\Gamma_{cb}^2 + 6Y(0)\right)$$

$$\delta c_{d=3} = -\frac{3\pi^4}{10}\left(\frac{T}{\varepsilon_F}\right)^3 \int_{T/\varepsilon_0}^{1} \left(\Gamma_{cb}^2 + 6Y(\theta)\right)\frac{d\theta}{\theta}$$

Charge excitations

Spin excitations

The function Y is not universal and depends on the cutoff.

$$Y(\theta) = \frac{2\Gamma_b^2}{X^2}\int_0^X \frac{\ln(1+t)}{(1+t)^2}\ln\left(\frac{X}{t}\right)dt$$

$$X = -\alpha_d \Gamma_b \ln\left[\max\{\theta, T/\varepsilon_0\}\right]$$

$$Y(\theta) = \Gamma_b^2\left(\frac{1}{2} - \frac{10}{9}X\right)$$ for $X \le 1$

$$Y(\theta) \simeq \frac{2\Gamma_b^2 \ln X}{X^2}$$ for $X \ge 1$

$$Y_{X=0} = \Gamma_b^2/2$$ ⇒ Agreement with:

Chubukov, Maslov, Gangadharaiah, Glazman (2005) for 2d,

Chubukov, Maslov, Millis (2005) for 3d

(Direct diagrammatic expansion)

What about $\gamma_1(\xi) = \frac{1}{\xi_f^* - \xi}$?

Does this instability mean anything?

Yes!

Formation of an order parameter Q(u):

$$Q_{\alpha\beta}(u) = <\psi_\alpha \bar{\psi}_\beta>$$

$$0 < u < 1$$

$$Q_{\alpha\beta} = -Q_{\beta\alpha} \Rightarrow Q_\gamma = \varepsilon_{\alpha\beta\gamma} Q_{\alpha\beta}$$

Equation for the “order parameter”

$$Q_0(u) = T\sum_{\omega}\int \frac{2\pi\bar{\Gamma}u^2 v_F Q_0(u)}{(v_F k_{\|})^2 + \omega^2 - Q_0^2(u)}\frac{dk_{\|}}{2\pi}$$

$$Q_0(u) = \Delta_0(u)\, qV\Lambda V^{-1}$$

$$q = zq_0 z^t, \; zz^t = 1$$

$$\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$1 = \pi\bar{\Gamma}u^2 T \sum_{|\omega|<\varepsilon_0} \frac{1}{\sqrt{\omega^2 + \Delta_0^2(u)}}$$

$$\omega = 2\pi nT, n = 0, \pm, \pm 2, \pm 3 \ldots$$

$\Delta_0(u)$ is a function of u, 0<u<1

Asymptotic behavior

$$\Delta_0(u) = \varepsilon_0 \exp\left(-\frac{1}{\bar{\Gamma} u^2}\right)$$

Low temperatures

$$\Delta_0(u) = \frac{\pi T \bar{\Gamma} u^2}{1 - \bar{\Gamma} u^2 \ln(\varepsilon_0 / T)}$$

High temperatures

$\Delta_0(u)$ is always non-zero!

Change of the specific heat at low temperatures

$$\delta C \propto -\frac{T}{\ln(\varepsilon_F / T)}$$

⟺ Universal

Interpretation: $\Delta_0(u)$ is the inverse time of the relaxation of the spin excitations.

Conclusions.

There can be a non-trivial life in the Fermi gas with a repulsion due to spin excitations.

All these effects should become very pronounced near quantum critical points like normal metal-magnetic states transitions (may be, in the high T_c cuprates(?))