2D SNS junction with Rashba spin-orbit interaction

Ol'ga V. Dimitrova and M. V. Feigel'man

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- Experimental S-2DEG-S junction geometry and the theoretical model
- Asymmetry of the quantum well and the Rashba interaction
- Generalization of the Beenakker's formula for the Andreev levels
- Spin-splitting of the Andreev levels
- Insensitivity of the Josephson current to the Rashba coupling

Experimental S-2DEG-S junction geometry

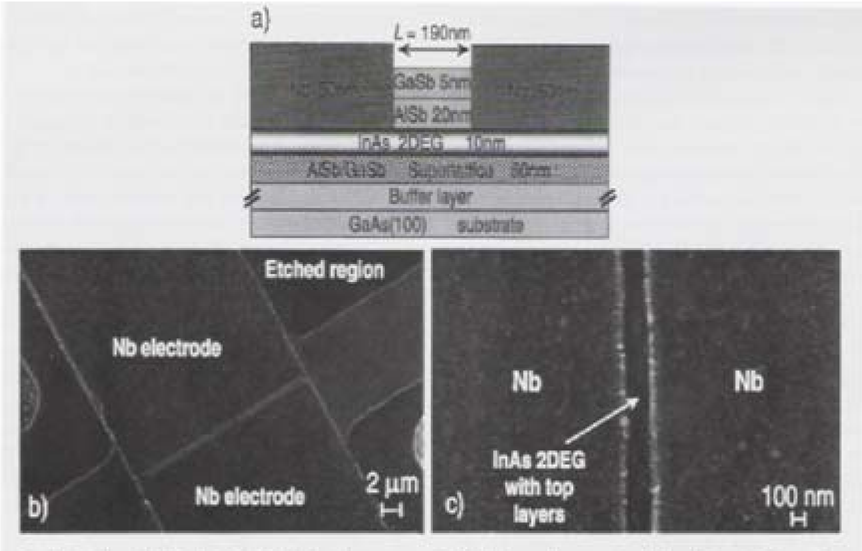


Fig. 1. (a) Sketch of the Nb/InAs-2DEG/Nb microstructure. The Nb-electrode separation is $L = 190$ nm; (b) Scanning electron micrograph of the device. The mesa lateral arms were used as additional probes for electrical characterization, and (c) Magnified view of Fig. 1(b) showing the semiconductor channel separating the two Nb contacts.

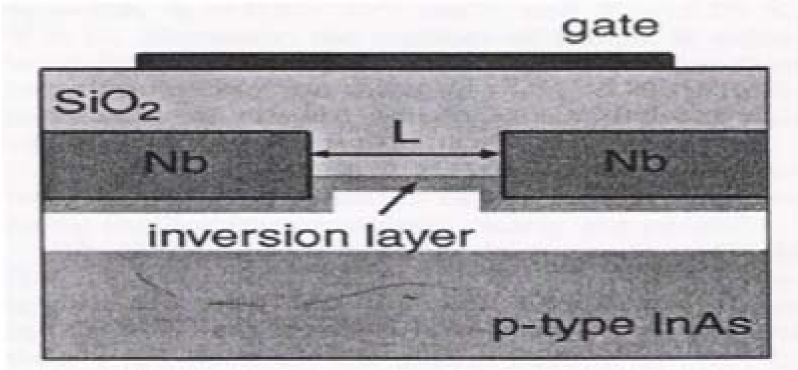
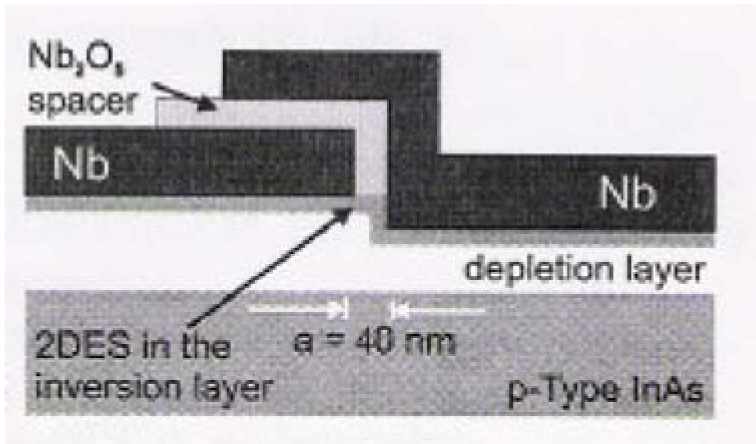
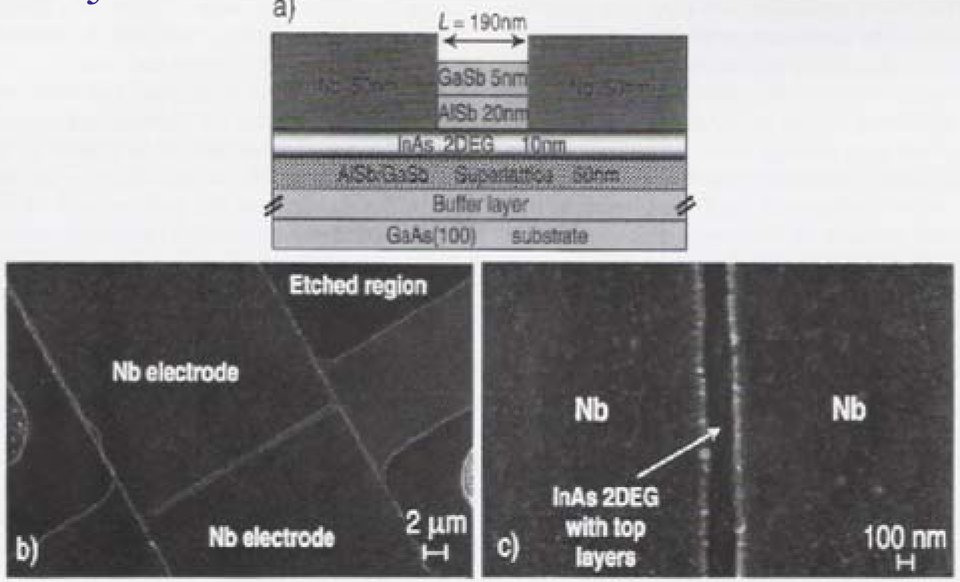


Fig. 1. Schematic cross-section of the Josephson junction.



F. Giazotto et al., *Journal of Superconductivity: Incorporating Novel Magnetism, Vol. 17, No. 2, April 2004*

Josephson current in Nb/InAs/Nb Highly Transmissive Ballistic Junctions



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- Two-dimensional electron gas formed at an InAs/AlSb heterojunction

  carrier concentration $n_{2DEG} = 7.4 \cdot 10^{11} \mathrm{cm}^{-2}$

  mobility $\mu = 75000 cm^2/Vs$

  effective mass $m_n = 0.036 m_e$

- Ballistic analysis of the system $(l_m > L)$, and in the clean limit $(\xi_N(T \geq 0.8K) < l_m)$:

  separation between electrodes $L = 190nm$

  mean free path $l_m = 1.1 \mu m$

  thermal coherence length $\xi_N(T) = 0.85 \mu m/T$

- High Intrinsic interface transmissivity $\sim 80\%$

- Superconducting gap in Nb:

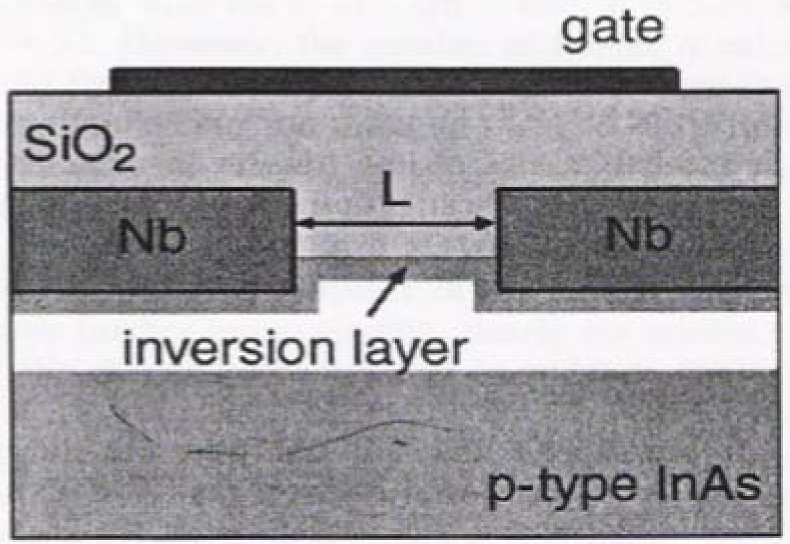
  $\Delta_{\mathrm{Nb}} = 1.4 meV$

- Characteristic product

  $I_c R_N = 165 \mu V$

M. Gracjar et al., *Physica C 372-376 (2002) 27-30*

Current-phase relation in Nb/InAs(2DEG)/Nb Josephson junction



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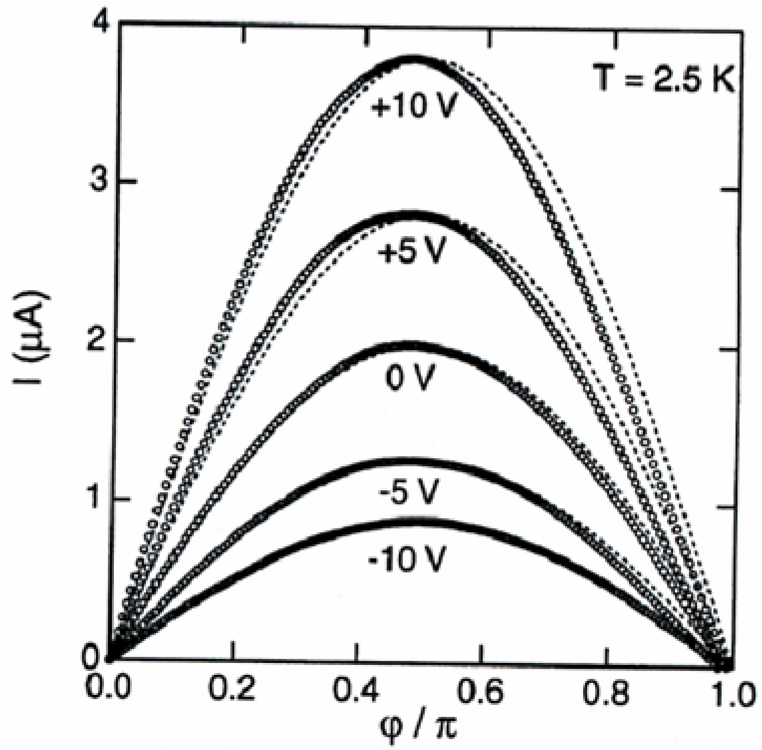
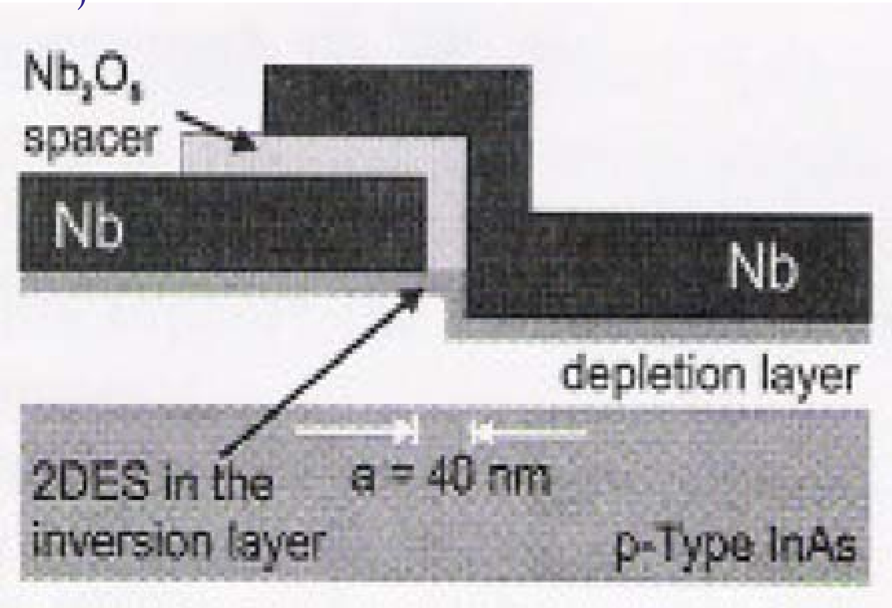


Fig. 3. CPRs for various gate voltages at $T = 2.5$ K. Dashed lines show sinusoidal CPR.

Mark Ebel et al. *cond-mat/0407206*

Current-phase relation in a Nb/InAs(2DEG)/Nb Josephson junction



High interface transperancy, deviation from sinusoidal CPR:

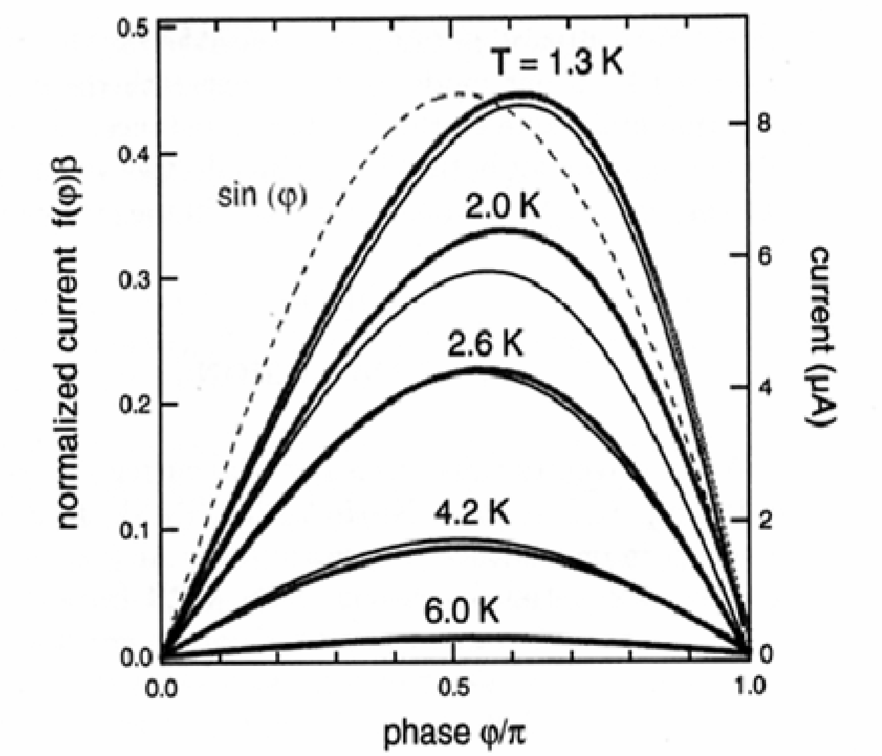


FIG. 3: CPR (red) at temperatures between 1.3 K and 6.0 K,

Theoretical models which were usually used

- All kind of ballistic contacts (long, short); disordered contacts (Chrestin,…, Kulik Omelyanchuk)
- Electron-electron interaction almost never taken into account (except paper of Aslamazov, Larkin, Ovchinnikov 1968)
- No Rashba interaction, except I. V. Krive(2004), E. Bezuglyi(2002)

Asymmetry of the quantum well and the Rashba interaction

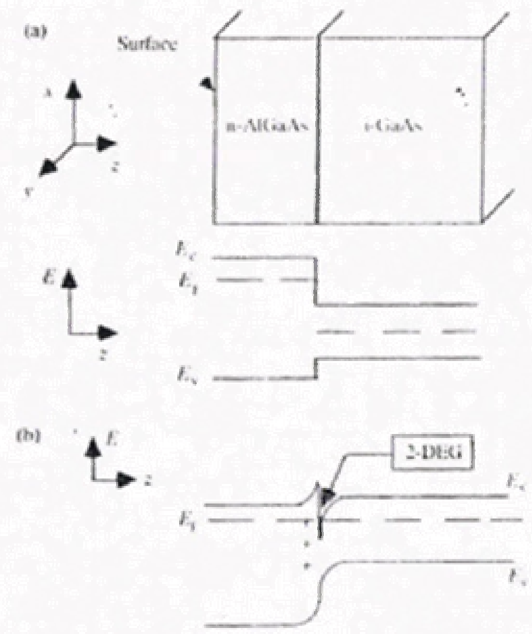


Fig. 1.1.1. Conduction and valence band line-up at a junction between an n-type AlGaAs and intrinsic GaAs, (a) before and (b) after charge transfer has taken place. Note that this is a cross-sectional view. Patterning (as shown in Fig. 0.3) is done on the surface (x–y plane) using lithographic techniques.

The symmetry-allowed linear splitting is represented as an added 'Rashba' term in the Hamiltonian:

$$\mathcal{H}_R = \alpha[\mathbf{p} \times \hat{\sigma}] \cdot \mathbf{n}$$

where $\hat{\sigma}$ is the vector of the Pauli spin matrices, $\mathbf{n}$ a unit vector perpendicular to the plane of the 2DEG, and α a parameter describing a strength of the spin-orbit splitting.

Spin splittings for two quantum well systems:

System	electron density 10^{12}cm^{-2}	Rashba splitting $2\alpha p_F$ (meV)
AlGaAs/GaAs	n=0.2-0.7	0.03-0.04
AlSb/InAs/AlSb	n= 1.2	3-15

Hamiltonian of the SNS system

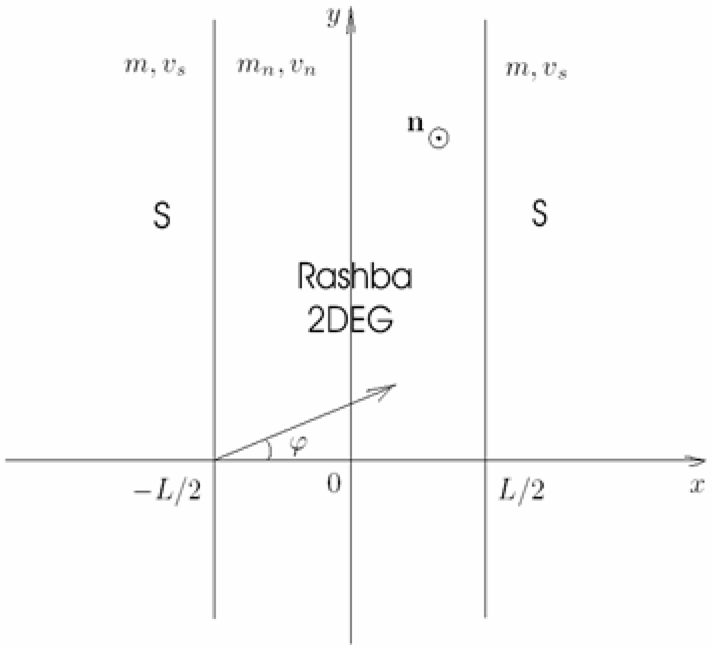


FIG. 1: Two-dimensional model of a superconductor/Rashba 2DEG/superconductor Josephson junction infinite in direction perpendicular to the current (along y axis). The Rashba 2DEG region has thickness L; m/m_n is the effective mass and v_s/v_n is the Fermi velocity in the S/2DEG; φ is the angle between the velocity direction of a quasiparticle and the x axis in the 2DEG region; $\mathbf{n}$ is a unit vector normal to the plane of the 2DEG.

Hamiltonian of the superconductors:

$$\hat{H}_S = \sum_{\mathbf{p}} \Psi_{\mathbf{p}}^{\dagger} \begin{pmatrix} \mathcal{H}_0(\mathbf{p}) & \Delta \\ \Delta^* & -\mathcal{H}_0(-\mathbf{p}) \end{pmatrix} \Psi_{\mathbf{p}}$$

$$\mathcal{H}_0 = \frac{\mathbf{p}^2}{2m} - E_{Fs}$$

Hamiltonian of the Rashba 2DEG:

$$\hat{H}_{2\mathrm{DEG}} = \sum_{\mathbf{p}} \Psi_{\mathbf{p}}^{\dagger} \begin{pmatrix} \mathcal{H}_R(\mathbf{p}) & 0 \\ 0 & -\mathcal{H}_R(-\mathbf{p}) \end{pmatrix} \Psi_{\mathbf{p}}$$

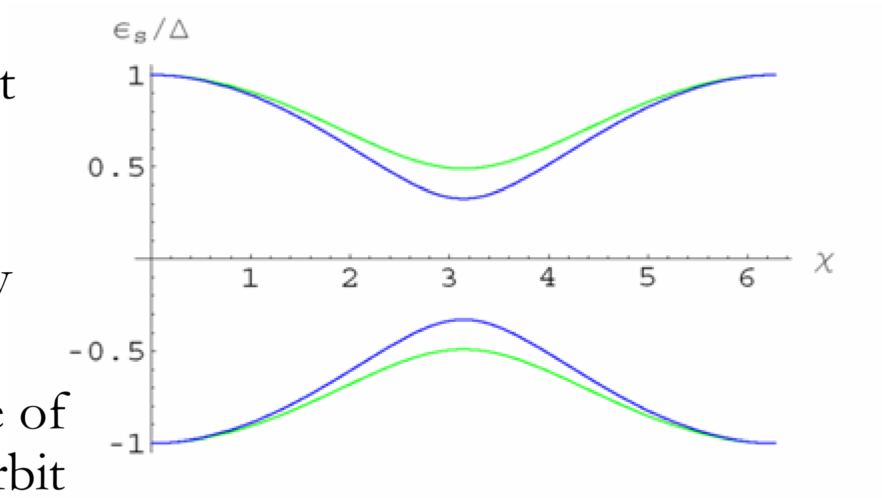
$$\mathcal{H}_R = \frac{\mathbf{p}^2}{2m_n} - E_{Fn} + \alpha[\mathbf{p} \times \hat{\sigma}] \cdot \mathbf{n}$$

Matching of the wave functions at the S/N boundaries:

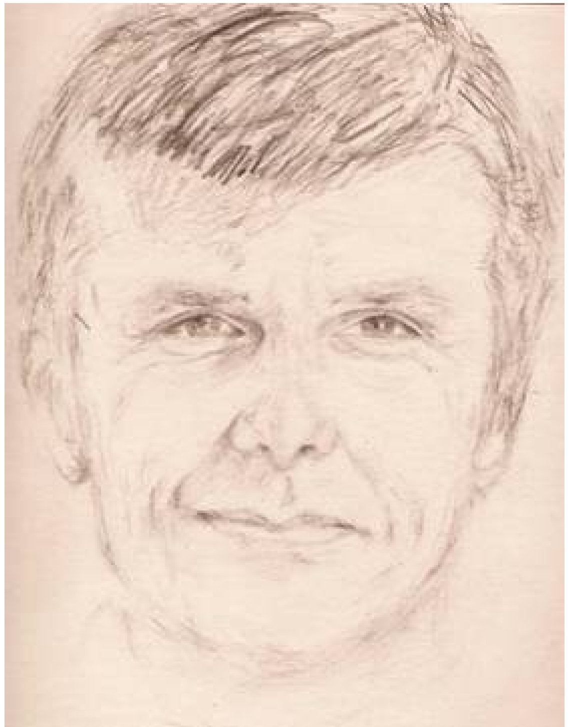
$$\Psi|_S^N = 0, \qquad \frac{1}{m}\partial_x \Psi|_S^N = 0$$

Main results

- Andreev energy is determined via the Transmission probabilities, and for a short junction the relation is derived explicitly
- Spin-splitting of the Transmission probabilities, leading to spin-split Andreev levels
- Independence of the semiclassical average of the total Josephson current on the spin-orbit coupling, derived for the general case of an arbitrary length of the contact and arbitrary Fermi-velocity mismatch, but without taking into account the electron-electron interaction



Equation which relates the excitation spectrum of the Josephson junction (the Andreev levels E) to the scattering matrix in the normal state S was derived by Carlo W. J. Beenakker in 1991



Equation for Andreev levels E:

$$\det[1 - r_{he} S_e(E) r_{eh} S_h(E)] = 0$$

Beenakker, 1991

$r_{he} = \gamma r_A$, $r_{eh} = \gamma r_A^*$

→ Andreev reflection matrix for e→h scattering

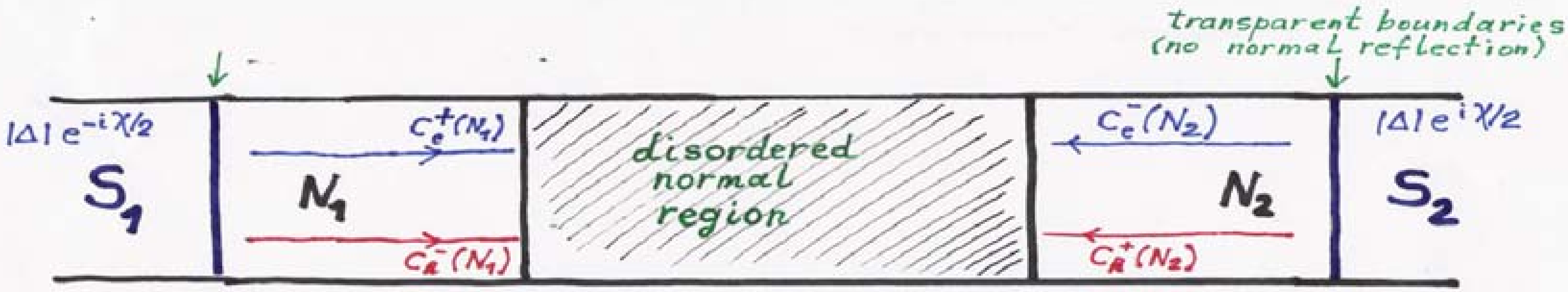
$$\gamma = e^{-i \arccos(E/\Delta)}$$

$$r_A = \begin{pmatrix} e^{i\chi/2} & 0 \\ 0 & e^{-i\chi/2} \end{pmatrix}$$

$S_{e(h)}(E)$ - electron (hole) scattering matrix

(which is trivial in spin space, if no spin-orbit interaction is present)

Derivation of the Beenakker formula – the condition of quantization of the discrete spectrum



$S_{1,2}$ – superconductor (left, right)

$N_{1,2}$ – ideal (impurity free) normal leads

$C_N^{in} = (c_e^+(N_1),\ c_e^-(N_2),\ c_h^-(N_1),\ c_h^+(N_2))$

wave, incident on the disordered normal region

$$C_N^{out} = \check{S}_N\, C_N^{in},$$

$$\check{S}_N = \begin{pmatrix} \check{S}_e(\epsilon) & 0 \\ 0 & \check{S}_h(\epsilon) \end{pmatrix}, \quad \check{S}_e(\epsilon) = \begin{pmatrix} \hat{R}_1(\epsilon) & \hat{T}_2(\epsilon) \\ \hat{T}_1(\epsilon) & \hat{R}_2(\epsilon) \end{pmatrix}$$

↳ S matrix of the normal region

$\varepsilon < |\Delta| \rightarrow$ no propagating modes in $S_1, S_2 \Rightarrow$

$$\Rightarrow \quad C_N^{in} = \check{S}_A\, C_N^{out},$$

$$\check{S}_A = \gamma \begin{pmatrix} 0 & r_A \\ r_A^* & 0 \end{pmatrix}, \quad r_A = \begin{pmatrix} e^{i\chi/2} & 0 \\ 0 & e^{-i\chi/2} \end{pmatrix}$$

↳ S matrix for Andreev reflection

(we neglected normal reflection at S/N interface)

The condition of bound states $C_N^{in} = \check{S}_A \check{S}_N C_N^{in}$ implies $\boxed{\mathrm{Det}[1 - \check{S}_A \check{S}_N] = 0}$

Generalization of the Beenakker's formula for Andreev levels in presence of Rashba spin-orbit interaction

In the presense of spin-orbit interaction the scattering matrix becomes spin-dependent, but still obeys time-reversal invariance ⇒

⇒

$SS^{\dagger} = 1$	$S^T(-p_y) = \hat{g}^T S(p_y)\hat{g}$	$S_h(E,p_y) = \hat{g}^T S_e^*(-E,-p_y)\hat{g}$
Unitarity	Time-reversal invariance	Due to special symmetry of BdG

⇒

$$\det\left[\frac{1}{\gamma} g^T S_e^*(E,p_y)\, g\, r_A^* - \gamma\, r_A^*\, g^T S_e^*(-E,p_y)\, g\right] = 0$$

equation for Andreev levels in the presence of spin-orbit interaction for any length of the contact (scattering matrix $S_e(E)$ is spin-, and energy-dependent)

⇒ in the short junction limit $\Delta \ll \hbar v_F/L$ we neglect the energy dependence in the scattering matrix S_e ⇒

⇒ four non-degenerate Andreev levels

$$E_{s,\eta} = \eta\Delta\sqrt{1 - \mathcal{T}_s(p_y)\sin^2\frac{\chi}{2}}$$

$\eta = \pm$

$s = \pm$

The role of boundary conditions on transmission eigenvalues

Boundary conditions in y direction

(w.f. of the normal layer $\Psi = \psi_1(x)\,\psi_2(y)\,\chi$)

Periodic, $\psi_2(0) = \psi_2(L_y)$,

$\psi_2(y) = e^{ip_y y}$, $\quad p_{y,n} = \frac{2\pi n}{L_y}$, $n = 0, \pm 1, \pm 2$

Closed, $\psi_2(0) = \psi_2(L_y) = 0$

$\psi_2(y)\big|_{\alpha=0} = \sin p_y y$, $\quad p_{y n} = \frac{\pi n}{L_y}$, $n = 1, 2, \dots$

Kramers theorem for transmission eigenvalues (T_s)

Periodic: not applicable, due to complex basis: $\psi_2^*(y) \neq \psi_2(y)$

T_s are spin-splitted ($T_+ \neq T_-$) (in presence of Rashba interaction)

Closed: applicable, due to real basis $\psi_2^*(y) = \psi_2(y)$

T_s are spin-degenerate

J. Phys. I 1 (1991) 493-513
Pier A. Mello, J.-L. Pichard

The number of the T_s levels as a function of the length of the contact L_y, the boundary conditions, and the presense of Rashba interaction

Periodic ($\alpha = 0$):

$p_y = 0$ ═══

$\pm \frac{p_F}{2}$ ═══ } $4 = 2\times 2$

$\pm p_F$ ═══

Rashba →

Periodic ($\alpha \neq 0$):

$p_y = 0$ ═══ 2 (spin)

$\pm \frac{p_F}{2}$ { ═══ $s=+1$, ═══ $s=-1$ } $\sim \alpha$

$\pm p_F$ { ═══ $s=+1$, ═══ $s=-1$

$N^{periodic}(\alpha=0) = \frac{p_F L_y}{2\pi} + 1$

Closed ($\alpha = 0$):

$n=1$ ═══ 2 spin

$n=2$ ═══

$n=3$ ═══

$n=4$ ═══

Rashba →

Closed ($\alpha \neq 0$):

$n=1$ ═══ 2 spin

$n=2$ ═══

$n=3$ ═══

$n=4$ ═══

$N^{closed}(\alpha=0) = N^{closed}(\alpha\neq 0) = \frac{p_F L_y}{\pi}$

Spin-orbital effect on Andreev levels:

spin-splitting, four non-degenerate A.l.

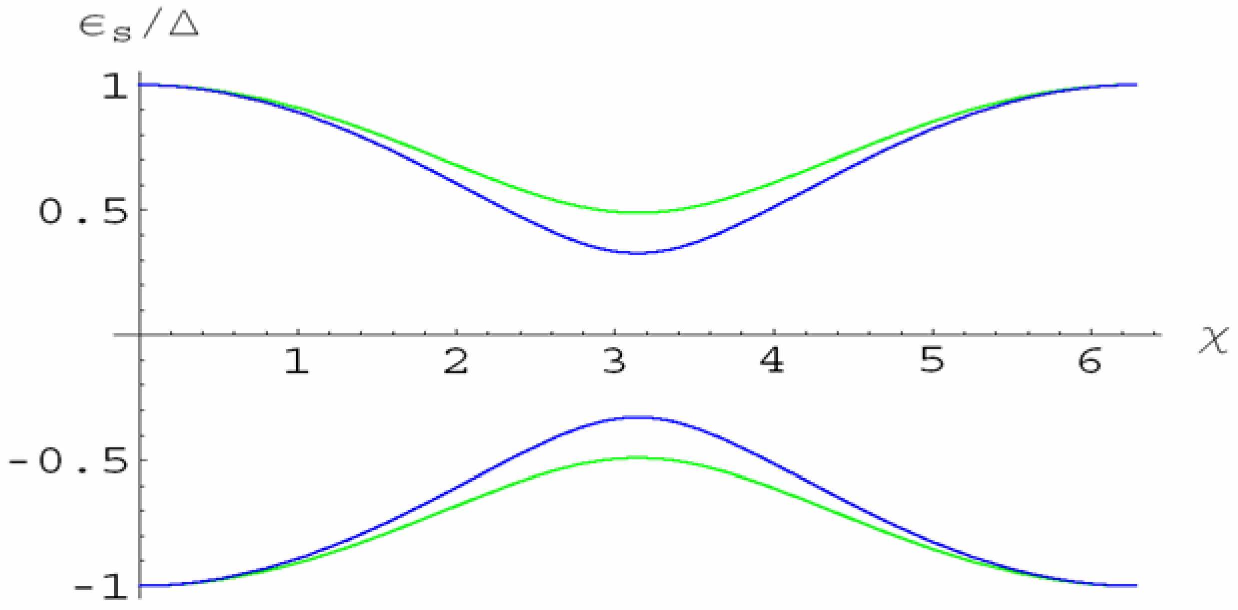


FIG. 3: The four spin-splitted Andreev levels $\pm\epsilon_s$, $s = \pm 1$, as a function of the superconducting phase difference χ, plotted for a value of the angle of propagation $\varphi = \pi/5$, and for a realistic S-2DEG-S junctions with parameters $v_s = 7 \cdot 10^7$cm/s, $v_n = 5 \cdot 10^7$cm/s, $\alpha \approx 0.2 \cdot 10^7$cm/s, $m = m_e$, $m_n = 0.035 m_e$, $L = 190$nm.

Scattering matrix S of S-Rashba 2DEG-S junction in the normal state

$\mathcal{T}_s(p_y)$ are transmission probabilities (eigenvalues of $\hat{T}^+\hat{T}$), depending on spin index $s=\pm$, and conserved momentum p_y.

⇑ translational invariance along y-direction

⇓ scattering channels, characterized by complex eigenfunctions $\propto e^{ip_y y}$

Specific model:

(1) 2DEG with Rashba interaction

$$H_R = \alpha[\vec{\sigma} \times \vec{p}] \cdot \vec{n}$$

(2) model of S/N boundaries:
normal reflection due to Fermi velocity mismatch

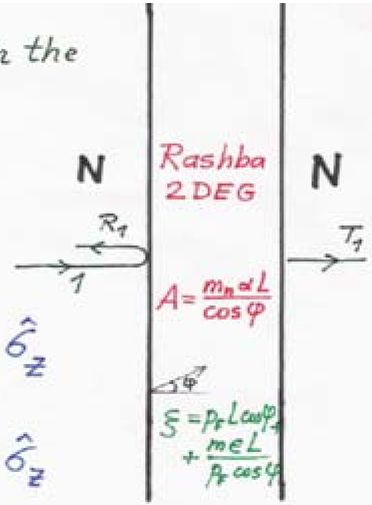
$$\vec{r}_1 = \frac{w-1}{w+1} \qquad \vec{t}_1 = \frac{2}{w+1}, \qquad w = \frac{v_{nx}}{v_{sx}}$$

$$\overleftarrow{r}_2 = \vec{r}_1, \quad \overleftarrow{t}_2 = \vec{t}_1 \qquad v_{nx} = v_n \cos\varphi$$

$$\overleftarrow{r}_1 = \vec{r}_2 = -\vec{r}_1, \qquad \overleftarrow{t}_1 = \vec{t}_2 = \frac{2w}{w+1} \qquad v_{sx} = \sqrt{v_s^2 - \left(\frac{m_n}{m}\right)^2 v_n^2 \sin^2\varphi}$$

$\check{S}$-matrix in the normal state

$$\check{S} = \begin{pmatrix} \hat{R}_1 & \hat{T}_2 \\ \hat{T}_1 & \hat{R}_2 \end{pmatrix}$$



$$\hat{T}_1 = T_0 + T_1\hat{\sigma}_x + T_3\hat{\sigma}_z$$

$$\hat{T}_2 = T_0 + T_1\hat{\sigma}_x - T_3\hat{\sigma}_z$$

$$T_0 = t\ sh(x - i\xi)\cos A$$

$$T_1 = -it\ ch(x - i\xi)\sin A \sin\varphi$$

$$T_3 = it\ sh(x - i\xi)\sin A \cos\varphi$$

$$x = \log\frac{1+w}{1-w} \qquad t = \frac{sh\, x}{sh^2(x - i\xi) + \sin^2 A \sin^2\varphi}$$

Spin-orbital effect on Transmission probabilities: spin-splitting

$$\mathcal{T}_\pm = \frac{\mathrm{sh}^2 x}{\mathrm{sh}^2 x + \sin^2(\xi \pm \beta/2)} \qquad \mathcal{T}_+(p_y) \neq \mathcal{T}_-(p_y)$$

$x = x(\varphi) = \log \frac{1+w(\varphi)}{1-w(\varphi)}$ → intrinsic interface parameter

$\xi = p_F L \cos\varphi$ → main semiclassical phase;

$A = m\alpha L/\cos\varphi$ → additional phase due to spin rotation by Rashba coupling.

$\cos\beta = 1 - 2\sin^2\varphi \sin^2 A.$

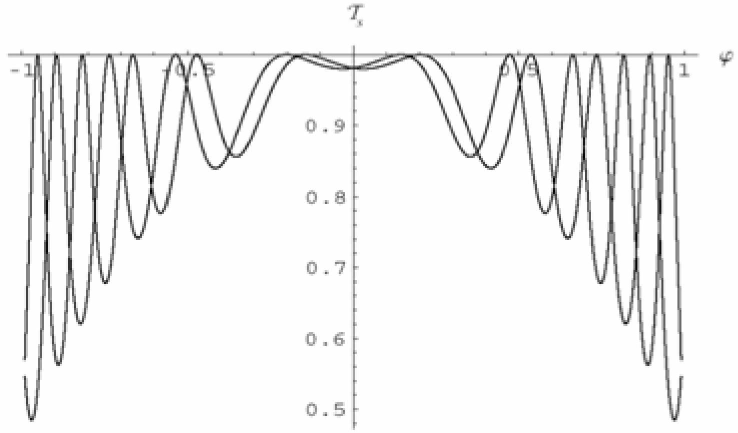


FIG. 2: The spin-splitted transmission eigenvalues $\mathcal{T}_s$, $s = \pm 1$, as functions of the angle φ of propagation of the quasiparticles inside the 2DEG, plotted for a realistic S-2DEG-S junction with parameters $v_s = 7 \cdot 10^7$cm/s, $v_n = 5 \cdot 10^7$cm/s, $m = m_e$, $m_n = 0.035 m_e$, $m_n \alpha/\hbar = 5 \cdot 10^4$cm^{-1}, $L = 190$nm. For these parameters and for value of the superconducting gap $\Delta = 1.4$meV: (1) the length of the contact L is shorter than the coherence length, $\xi_0 = \hbar v_s/\Delta = 330$nm; (2) the Rashba velocity is much smaller than the Fermi velocity in the 2DEG, $\alpha/v_n \approx 0.03$; (3) the system is within the semiclassical limit, $p_F L/\hbar = m_n v_n L/\hbar \approx 30$; (4) the spin-orbital splitting $2\alpha p_F \approx 3.3$meV is larger than the superconducting gap Δ; (5) the S/N boundaries are almost transparent ($v_s/v_n \approx 1.4$), which means a large experimental value of the critical current.

Josephson current: independence on Rashba (short junction)

$$I(\chi) = \frac{e\Delta^2}{2\hbar}\sin\chi \int \frac{L_y\, dp_y}{2\pi\hbar} \sum_{s=\pm} \frac{\mathcal{T}_s(p_y)}{E_{s,+}(\chi)}\, th\frac{E_{s,+}(\chi)}{2T}$$

(short contact limit)

In the semiclassical limit $p_F L \to \infty$ the average Josephson current with the use of distribution function $\mathcal{P}_\varphi(\mathcal{T})$:

$$I(\chi) = \frac{e\Delta}{2\hbar}\int_{-\pi/2}^{\pi/2} \frac{d\varphi \cos\varphi}{2\pi} \int \mathcal{P}_\varphi(\mathcal{T})\, d\mathcal{T}\, \frac{\mathcal{T}\sin\chi}{\sqrt{1-\mathcal{T}\sin^2\frac{\chi}{2}}}\, th\frac{\Delta\sqrt{1-\mathcal{T}\sin^2\frac{\chi}{2}}}{2T}$$

Universality of distribution function for the transmission probabilities:

$$\mathcal{P}_\varphi(\mathcal{T}_\pm) = \int \delta(\mathcal{T}-\mathcal{T}_\pm(\xi))\, d\xi = \frac{th\, x}{2\mathcal{T}\sqrt{1-\mathcal{T}}\sqrt{\mathcal{T}-th^2 x}}$$

$\Downarrow$

independence of the average semiclassical Josephson current on spin-orbit coupling.
(short contact limit)

Spin-splitted Andreev levels at an arbitrary length of the contact

(obtained via direct matching of the wavefunctions obeying BdG equations in the 2DEG and in both superconductive regions)

Equation for Andreev levels E:

$$\boxed{\cos 2\xi = f_\pm(E)},$$

$$f_\pm(E) = Q\cos\beta \pm \sqrt{1-Q^2}\,\sin\beta$$

$$Q = \cos\psi + \frac{4k^2K^2\Delta^2(\cos\psi + \cos\chi)}{(K^2-k^2)^2(\Delta^2-E^2)}$$

$k = p_F\cos\varphi$,

$K = \sqrt{\mathcal{P}_F^2 - p_F^2\sin^2\varphi}$;

$$\psi = 2\,\mathrm{Arctg}\,\frac{2kKE}{(K^2+k^2)\sqrt{\Delta^2-E^2}} + \mathcal{E}$$

$$\mathcal{E} = \frac{2m_* EL}{k}$$ – energy dependence of the quasiclassical phase

$\Rightarrow$ Andreev levels are spin-splitted for any length of the contact in the presence of Rashba interaction.

Total Josephson current for the junction of an arbitrary length: independence on Rashba

We use the relation between the total Josephson current $I_{total}(\chi)$, and the spectral function $g(\epsilon,\chi)$:

$$I_{total}(\chi) = L_y \frac{4e}{\hbar} T \sum_{s=\pm} \int \frac{dp_y}{2\pi\hbar} \sum_{\omega_n>0} \partial_\chi \ln g_s(i\omega_n,\chi),$$

presence of spin-splitting ($\sum_{s=\pm}$)

continuous scattering channels are characterized by the transverse momentum p_y ($\int dp_y$)

Н. М. Щелкачёв, PhD thesis.

where $g(\epsilon,\chi) = \det[1 - S_h r_{he} S_e r_{eh}] = g_+(\epsilon,\chi)\, g_-(\epsilon,\chi)$,

$$g_s(\epsilon,\chi) = \cos 2\xi - f_s(\epsilon,\chi)$$

Within the semiclassical limit, after $\frac{1}{\pi}\int_0^\pi d\xi \ldots$,

$$I_{total}(\chi) = -L_y p_F \frac{4e}{\hbar^2} T \sum_{\omega_n>0} \int_{-\pi/2}^{\pi/2} \frac{d\varphi}{2\pi} \cos\varphi \frac{\partial_\chi Q}{\sqrt{|1-Q^2|}} \qquad Q = Q(i\omega_n,\chi)$$

Average Josephson current does not depend on Rashba coupling for any L/ξ_0.

Derivation of the relation between the total Josephson current **I(χ)** and the spectrum function **g(ε, χ)**

Free energy of the contact in mean-field approximation

$$\Omega(\chi) = \frac{1}{g}\int |\Delta(\vec{z})|^2 d^2\vec{z} - 2T \sum_{\nu, \varepsilon_\nu > 0} \ln 2\cosh\frac{\varepsilon_\nu}{2T} + \sum \int \psi_\nu^+(\vec{z}) \begin{pmatrix} \hat{\xi} & 0 \\ 0 & \hat{\xi}^* \end{pmatrix} \psi_\nu(\vec{z})\, d^2\vec{z}$$

$\psi_\nu(\vec{z}) = (u, v)^T$ - wave functions of the Bogolyubov quasiparticles, with dispersion ε_ν, obeying BdG equations

Josephson current

$$I(\chi) = -\frac{2e}{\hbar}\partial_\chi \Omega(\chi) = \frac{2e}{\hbar} T \sum_\nu \ln\cosh\frac{\varepsilon_\nu}{2T} =$$

$$= \frac{2e}{\hbar} T \int_C d\varepsilon \ln\left(\cosh\frac{\varepsilon}{2T}\right) \partial_\chi \rho(\varepsilon,\chi) = \frac{e}{i\pi\hbar}\int_C d\varepsilon\, \mathrm{th}\left(\frac{\varepsilon}{2T}\right) \partial_\chi \ln g(\varepsilon,\chi) = \frac{4e}{\hbar} T \sum_{\omega>0} \partial_\chi \ln g(i\omega, \chi)$$

$$\rho = -\frac{1}{i\pi}\mathrm{Tr}\left(G^R = \frac{1}{E - (H_0 + V) + i0}\right),$$

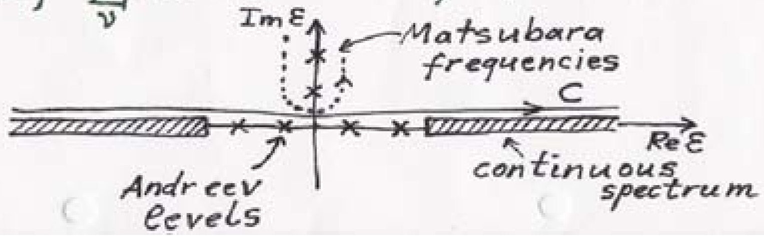
(H_0+V) – total Hamiltonian of the system

$\mathrm{Re}\,\rho = \sum_\nu \delta(\varepsilon - \varepsilon_\nu)$ – density of states

$\omega = 2\pi T(n + 1/2),\ n = 0, \pm 1, \ldots$

$$g = \det[1 - S_h r_{he} S_e r_{eh}] = \prod_{s=\pm}\left(\cos 2\xi - f_s(\varepsilon,\chi)\right)$$

$$\rho = -\frac{1}{\pi i}\partial_\varepsilon \ln g(\varepsilon,\chi)$$



$g(\varepsilon,\chi) = 0$ – equation on Andreev levels

Conditions for experimental observation of the Andreev levels

- Resonant absorbtion of microwaves
- Measurement of the tunneling density of states

Finite length of the S-2DEG-S in y direction L_y

1) $L_y \gg L_0 = \hbar / m_n \alpha \Rightarrow$ discrete set of transmission channels, $N_{ch} = 2L_y/\lambda_F$

2) $L_y \ll L_0, \quad \Rightarrow$ strong size quantization, $\hbar v_n / L_y \gg \alpha p_F, \Rightarrow$ suppression of the spin-orbital effects.

Open problems

- Calculation of the average spin polarization S_y in the 2DEG region, existing at a nonzero supercurrent in the SNS junction
- Taking into account of the electron-electron interaction: a supercurrent-induced average spin polarization will induce, in the presence of e-e interaction, an effective Zeeman field which may strongly modify the Andreev levels as well as the Josephson current