Have instantons been found experimentally?

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A.M.M.P., I.S.B. Phys. Rev. Lett. 95, 189701 (2005).
A.M.M.P., I.S.B. cond-mat/0502488.

Motivation

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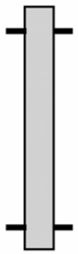
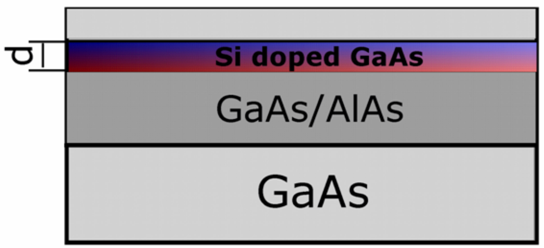
Topological Oscillations of the Magnetoconductance in Disordered GaAs Layers

S. S. Murzin,[1,4] A. G. M. Jansen,[2,4] and I. Claus[3,4]

S.S.Murzin, et.al., PRL 80, 2681 (1998)
S.S.Murzin et.al., PRB 59, 7330 (1999)

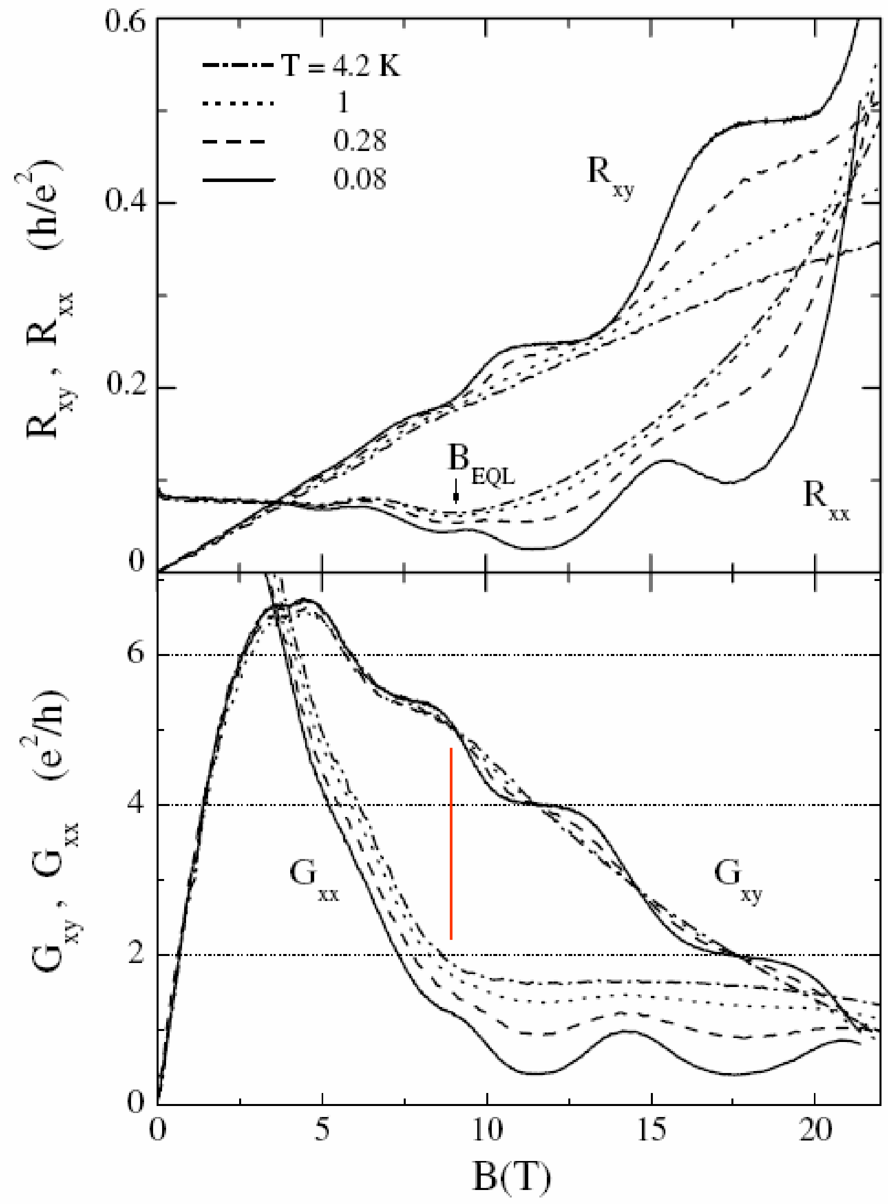




Si-doped n-type GaAs

n_{3D}(SdH at B<5 T)	$2\cdot 10^{17}$ cm^{-3}
n_{2D}(R_{xy} at B<3T)	$2\cdot 10^{12}$ cm^{-2}
d	100 nm
l(at B=0)	30 nm
μ	2.500 cm^2/V$\cdot$ s
$1/\tau$	>80 K

Motivation



- at $T=4.2K$
  $G_{jk}(T,B)$ are monotonic functions of B
- at $T\lesssim 1K$
  $G_{jk}(T,B)$ oscillate for $B>B_{EQL}$
- if one write

$$G_{jk} = G_{jk}^{sm}+G_{jk}^{osc}$$

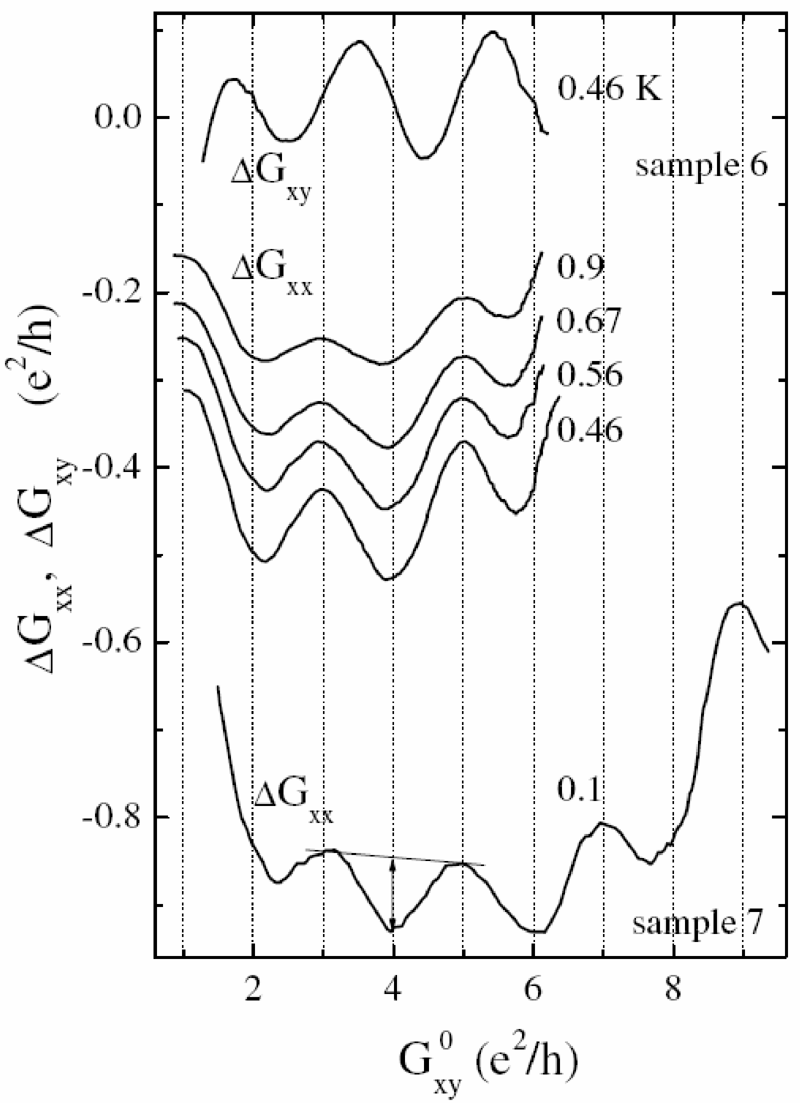
where

G_{jk}^{sm} is a smooth part
G_{jk}^{osc} is an oscillating part

then

$$\partial G_{xx}^{sm}/\partial B \approx 0$$
$$\partial G_{xy}^{sm}/\partial T \approx 0$$

Motivation



$B \Leftrightarrow G_{xy}^{0} = G_{xy}(T{=}4.2K,B)$

$G_{xx}^{0} = G_{xx}(T{=}4.2K,B)$

$G_{jk}(T,B) \Rightarrow G_{jk}(T,G_{xy}^{0})$

$\Delta G_{jk}(T,B)=G_{jk}-G_{jk}^{0}$

G_{xy}^{sm} is independent of $T \rightarrow$

$G_{xy}^{sm} = G_{xy}^{0}$

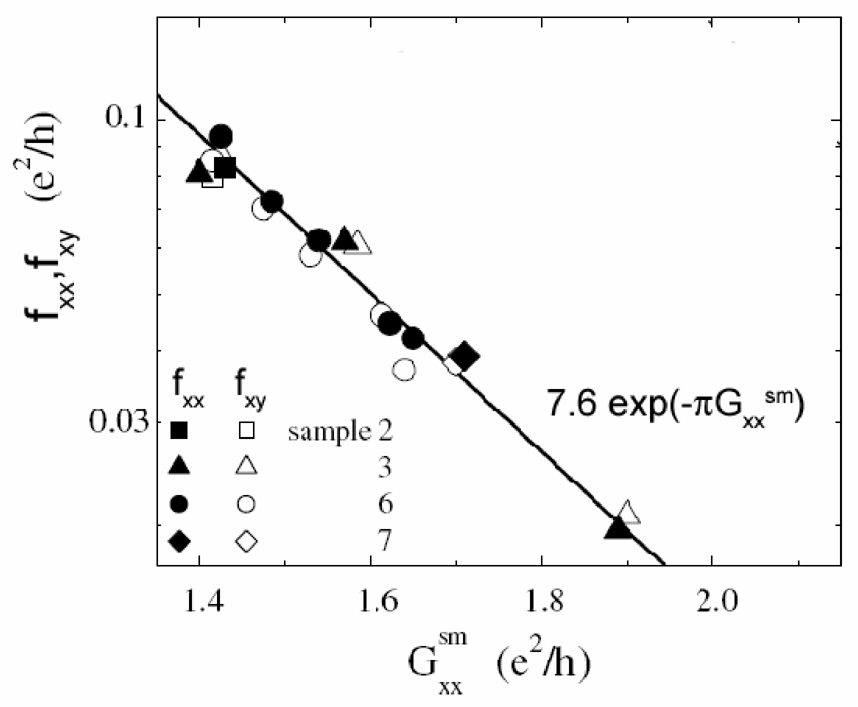
- ΔG_{xx} has extremum at $G_{xy}^{0} \approx 2,3,4,5,6$
- ΔG_{xy} has extremum at $G_{xy}^{0} \approx 3/2,5/2,7/2,9/2$

Motivation

- In the experiment

$$G_{xx}(T, G^0_{xy}) = G^{sm}_{xx}(T) - [f_{xx}(G^{sm}_{xx}(T)) - f_{xx}(G^0_{xx})] \cos \pi G^0_{xy},$$

$$G_{xy}(T, G^0_{xy}) = G^0_{xy} - [f_{xy}(G^{sm}_{xx}(T)) - f_{xy}(G^0_{xx})] \sin \pi G^0_{xy}$$



where

$$G^{sm}_{xx}(T) = G^0_{xx} + \frac{\lambda}{\pi} \ln T/T_0$$

with $\lambda \approx$ *0.8 – 1*, *T_0=4.2K* and

$$f_{jk}(g) \approx 7.6\, e^{-\pi g}$$

Model

- 2D electrons in a perpendicular magnetic field (electron spin is polarized)

Static vector potential

Trial vector potential

$$
\begin{aligned}
Z &= \int D[\psi,\psi^\dagger]\,\exp S[\psi,\psi^\dagger] \\
S &= \int_0^{1/T} d\tau \int d\mathbf{r}\,\psi^\dagger(\mathbf{r},\tau)\left[-\partial_\tau - \frac{1}{2m_e}\left(-i\nabla - \mathbf{A}^{\mathrm{st}} - \mathbf{A}\right)^2 - V(r) + \mu\right]\psi(\mathbf{r},\tau) \\
&- \frac{1}{2}\int_0^{1/T} d\tau \int d\mathbf{r} \int d\mathbf{r}'\,\psi^\dagger(\mathbf{r},\tau)\psi^\dagger(\mathbf{r}',\tau)U(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}',\tau)\psi(\mathbf{r},\tau)
\end{aligned}
$$

e-e interaction

Model

F.Wegner, 1979
K.B.Efetov, A.I.Larkin, D.E.Khmelnitskii, 1980
A.M.Finkelstein, 1983
A.M.M.Pruisken, 1984
A.M.M.Pruisken, M.A.Baranov, 1995
A.M.M.Pruisken, M.A.Baranov, B.Škorić, 1999

- Effective theory is nonlinear σ model with θ term

$$S[Q] = \frac{\sigma_{xx}}{8}\int d\mathbf{r}\,\mathrm{tr}[D_j,Q]^2 + \frac{\sigma_{xy}}{8}\int d\mathbf{r}\,\mathrm{tr}\,\varepsilon_{jk}Q[D_j,Q][D_k,Q]$$
$$+\ \pi T z\int d\mathbf{r}\left[\sum_{\alpha n}\mathrm{tr}\,I_n^\alpha Q\,\mathrm{tr}\,I_{-n}^\alpha Q + 4\,\mathrm{tr}\,\eta Q - 6\,\mathrm{tr}\,\eta\Lambda\right]$$

Q(r) is a matrix field in replica and Matsubara frequency spaces

$$Q^2 = 1$$

σ_{jk} is mean-field conductances

T is temperature

z is the singlet-interaction amplitude

Covariant derivative

$$D_j = \nabla_j - i\sum_{\alpha,n}(A_j)_n I_n^\alpha$$

Constant matrices

$$(I_n^\alpha)_{km}^{\beta\gamma} = \delta^{\alpha\beta}\delta^{\alpha\gamma}\delta_{k,n+m}$$
$$\eta_{nm}^{\alpha\beta} = n\delta^{\alpha\beta}\delta_{nm}$$
$$\Lambda_{nm}^{\alpha\beta} = \mathrm{sgn}(n)\delta^{\alpha\beta}\delta_{nm}$$

Model

- $\mathcal{F}$ –invariance (global symmetry)

$$Q \rightarrow WQW^{-1} \qquad W = \exp\left(i\sum_{\alpha,n} w_n I_n^\alpha\right)$$

- Gauge invariance

$$A_j \rightarrow A_j + \nabla_j w$$

$$Q \rightarrow W(r)QW^{-1}(r) \qquad W = \exp\left(i\sum_{\alpha,n} w_n(\mathbf{r}) I_n^\alpha\right)$$

Model

- Conductivity as linear response on a trial vector potential $A_n(q)$

$$\sigma'_{jk}(q, i\omega_n) = \epsilon_{jk}\sigma_{xy} - \delta_{jk}\frac{\sigma_{xx}}{4n}\left\langle \mathrm{tr}\,[I_n^\alpha, Q][I_{-n}^\alpha, Q]\right\rangle + \frac{\sigma_{xx}^2}{2nD}\int d\mathbf{r_2}\, e^{i\mathbf{q}(\mathbf{r_1}-\mathbf{r_2})}\epsilon_{aj}\epsilon_{bk}\langle \mathrm{tr}\, I_n^\alpha Q(\mathbf{r_1})\nabla_a Q(\mathbf{r_1})\mathrm{tr}\, I_{-n}^\alpha Q(\mathbf{r_2})\nabla_b Q(\mathbf{r_2})\rangle$$

- The singlet interaction amplitude

$$z' = -\frac{1}{2\pi L^2 \mathrm{tr}\,\eta\Lambda}\frac{\partial \ln Z}{\partial T} = -\frac{z}{2L^2\mathrm{tr}\,\eta\Lambda}\left\langle \int d\mathbf{r}\left[\sum_{\alpha n}\mathrm{tr}\, I_n^\alpha Q \mathrm{tr}\, I_{-n}^\alpha Q + 4\mathrm{tr}\,\eta Q - 6\mathrm{tr}\,\eta\Lambda\right]\right\rangle$$

Note: it was proven that $\sigma_{jk}(q=0, i\omega_n \to \omega \to 0)$ and z' are identically the same as one derives from the standard background field formalism.

Model

- Non-trivial topology due to the magnetic field B

$$S_{\text{top}}[Q] = 2\pi i \sigma_{xy} C[Q] = \frac{\sigma_{xy}(B)}{4} \int d\mathbf{r} \operatorname{tr} Q \nabla_x Q \nabla_y Q$$
$$\rightarrow -\frac{\sigma_{yx}(-B)}{4} \int d\mathbf{r} \operatorname{tr} Q \nabla_x Q \nabla_y Q$$

- $C[Q]$-integer provided spherical boundary conditions

$$Q\big|_{edge} = \Lambda$$

which are dynamically generated by the theory itself.

Model

- Instanton (O(3) instanton embedded into large Q matrix)

rotations

$$Q_{inst}(r) = U_0^{-1} R^{-1}(r) \Lambda R(r) U_0$$

replicas

frequencies

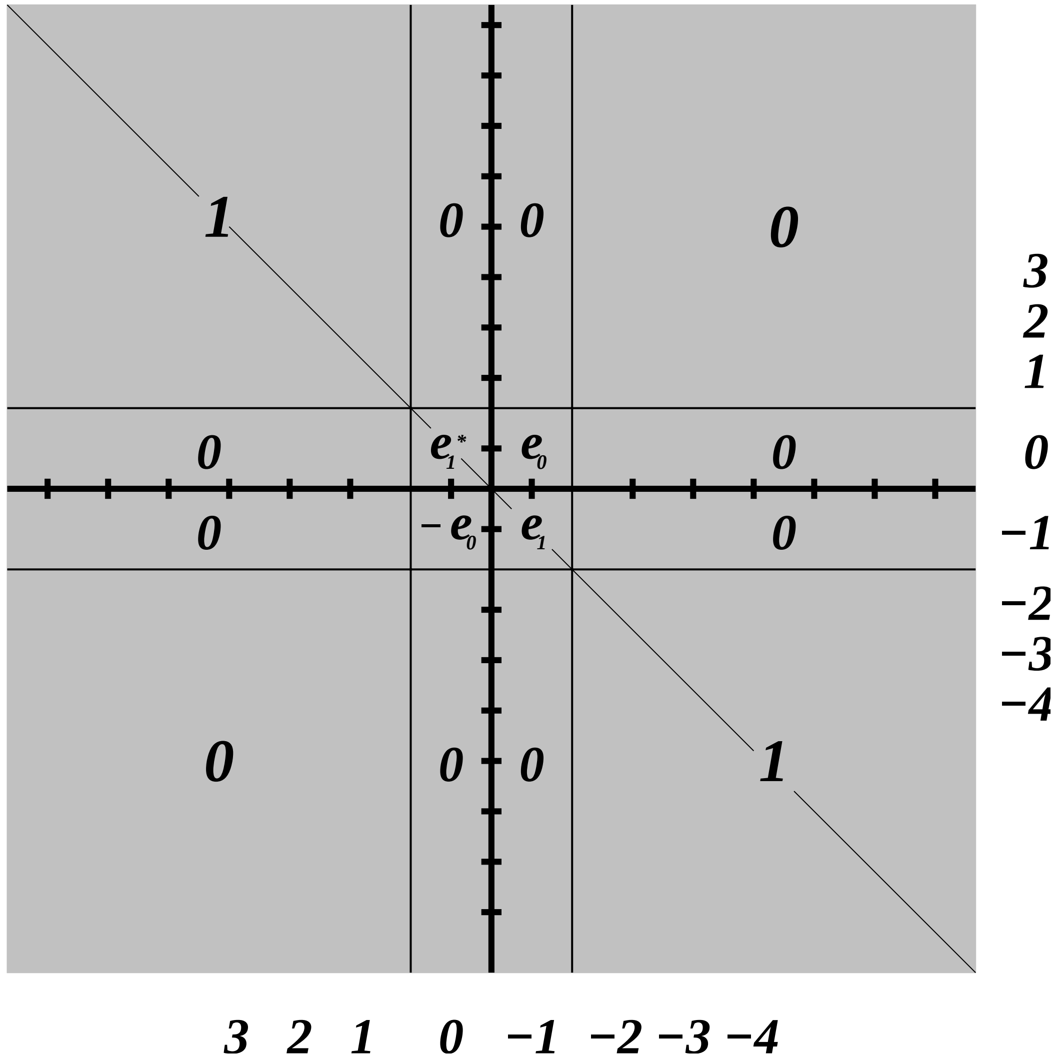
$$R_{nm}^{\alpha\beta} \propto \delta^{\alpha\beta}$$

size

$$e_0 = \frac{\lambda}{\sqrt{|z - z_0|^2 + \lambda^2}}$$

position

$$e_1 = \frac{z - z_0}{\sqrt{|z - z_0|^2 + \lambda^2}}$$



Renormalization group analysis

- Non-perturbative RG equations at *T=0*

$$\frac{d\sigma'_{xx}}{d\ln L} = \beta_\sigma = -\frac{2}{\pi} - \frac{b}{\sigma'_{xx}} - D\sigma'^2_{xx} e^{-2\pi\sigma'_{xx}} \cos 2\pi\sigma'_{xy}$$

$$\frac{d\sigma'_{xy}}{d\ln L} = \beta_H = -D\sigma'^2_{xx} e^{-2\pi\sigma'_{xx}} \sin 2\pi\sigma'_{xy}$$

$$\frac{d\ln z'}{d\ln L} = \gamma_z = -\frac{1}{\pi\sigma'_{xx}} - \frac{c}{\sigma'^2_{xx}} - \frac{D}{6}\sigma'_{xx} e^{-2\pi\sigma'_{xx}} \cos 2\pi\sigma'_{xy}$$

where

$b \approx 0.66$ *

$c = (3/\pi^2+1/6) \approx 0.47$

$D = 16\pi\, e^{1-4\gamma} \approx 13.6$

* analytic expression for *b* in M.A.Baranov, I.S.Burmistrov, A.M.M.Pruisken, 2002

Renormalization group analysis

- Conductances at finite temperature

$$G_{jk} = G_{jk}(Tz'l^2, \sigma'_{xx}, \sigma'_{xy})$$

where l is microscopic length at which σ_{jk}' and z' are defined

- G_{jk} should be independent of l (Callan-Symanzik equation)

$$\left[\beta_\sigma \frac{\partial}{\partial \sigma'_{xx}} + \beta_H \frac{\partial}{\partial \sigma'_{xy}} + (2+\gamma_z)\frac{\partial}{\partial \ln Tz'}\right] G_{jk} = 0$$

Renormalization group analysis

- The dependence of $G_{jk}(T)$ on σ_{xy}'

$$
\begin{aligned}
G_{xx}(T) &= g_0(Tz', \sigma'_{xx}) - g_\sigma(Tz', \sigma'_{xx}) \cos 2\pi\sigma'_{xy}, \\
G_{xy}(T) &= \sigma'_{xy} - g_H(Tz', \sigma'_{xx}) \sin 2\pi\sigma'_{xy}
\end{aligned}
$$

where g_0, g_σ and g_H obey

$$
\begin{aligned}
\left[\beta_\sigma^0 \frac{\partial}{\partial \sigma'_{xx}} + (2+\gamma_z^0)\frac{\partial}{\partial \ln Tz'}\right] g_0 &= 0 \\
\left[\beta_\sigma^0 \frac{\partial}{\partial \sigma'_{xx}} + (2+\gamma_z^0)\frac{\partial}{\partial \ln Tz'}\right] g_\sigma &= -D\sigma'^2_{xx} e^{-2\pi\sigma'_{xx}} \frac{\partial g_0}{\partial \sigma'_{xx}} \\
\left[\beta_\sigma^0 \frac{\partial}{\partial \sigma'_{xx}} + (2+\gamma_z^0)\frac{\partial}{\partial \ln Tz'}\right] g_H &= -D\sigma'_{xx} e^{-2\pi\sigma'_{xx}}
\end{aligned}
$$

Here β_σ^0 and γ_z^0 mean β_σ and γ_z with $D=0$.

Renormalization group analysis

- Scaling variable

$$X = T\bar{z}\xi^2$$

where the correlation length

$$\xi = l(\sigma'_{xx})^{-\pi^2 b/4} \exp\left(\frac{\pi\sigma'_{xx}}{2}\right)$$

and

$$\bar{z} = \frac{z'}{\sqrt{\sigma'_{xx}}}\left[1 + \pi\frac{2c - b)}{2\sigma'_{xx}}\right]$$

satisfies

$$\left[\beta^0_\sigma \frac{\partial}{\partial\sigma'_{xx}} + (2 + \gamma^0_z)\frac{\partial}{\partial \ln Tz'}\right] X = 0$$

Hence $g_0 = g_0(X)$ is arbitrary regular function of X !

Renormalization group analysis

- Temperature dependence of conductances

$$G_{xx}(T) = g_0 - \left[f_{xx}(g_0) - f_\infty(\sigma'_{xx})\right]\pi X g'_0 \cos 2\pi\sigma'_{xy},$$
$$G_{xy}(T) = \sigma'_{xy} - \left[f_{xy}(g_0) - f_\infty(\sigma'_{xx})\right]\sin 2\pi\sigma'_{xy},$$

where f_{xx} and f_{xy} are arbitrary regular functions and $f_\infty(\sigma'_{xx}) = \frac{D}{4}\sigma'^2_{xx}e^{-2\pi\sigma'_{xx}}$

- Smooth part of

$G_{xy}^{sm}(T)=\sigma_{xy}'$ has no dependence on temperature T

$G_{xx}^{sm}(T)=g_0(X)$ has no dependence on magnetic field (i.e. σ_{xy}')

Renormalization group analysis

- Temperature dependence of G_{xy}

$$G_{xy}(T) = G^0_{xy} - [f_{xy}(G^{sm}_{xx}(T)) - f_{xy}(G^{sm}_{xx}(T_0))] \sin 2\pi G^0_{xy}$$

where $G_{xy}{}^0 = G_{xy}(T_0)$ and $G_{xx}{}^{sm}(T)=g_0(X)$

- At $g_0 \gg 1$

$$g_0(X) = \frac{1}{\pi} \ln X$$

such that

$$G^{sm}_{xx}(T) = G^{sm}_{xx}(T_0) + \frac{1}{\pi} \ln T/T_0$$

Renormalization group analysis

- Temperature dependence of G_{xx}

$$G_{xx}(T) = \tilde{G}_{xx}(T) - [f_{xx}(G_{xx}^{sm}(T)) - f_{xx}(G_{xx}^{sm}(T_0))] \cos 2\pi G_{xy}^{0}$$

where $$\tilde{G}_{xx}(T) = G_{xx}(T_0) + \frac{1}{\pi} \ln T/T_0$$

- If we choose T_0 to be high enough then

$$G_{xx}(T_0) \equiv G_{xx}^{sm}(T_0)$$

$$\tilde{G}_{xx}(T) \equiv G_{xx}^{sm}(T)$$

Conclusions

• We predict "instanton" oscillations in magnetoconductance as experimentally verified by Murzin et.al.

• Experimentally $f_{xx}(g) \approx f_{xy}(g) \approx 7.6\, e^{-2\pi g}$ for $g \sim 1$

• The "instanton" oscillations of magnetoconductance are the direct evidence for the *existence* and *importance* of the instantons

Model

- Nonlinear σ model with θ term can be accurately derived for

I. weak magnetic field and large broadening of Landau levels

$$\mu \gg \omega_H \gg \tau^{-1} \gg T$$

$$\sigma_{xx} = 2\pi \frac{n_e \tau / m_e}{1 + \omega_H^2 \tau^2}, \qquad \sigma_{xy} = \sigma_{xx} \omega_H \tau$$

II. weak magnetic field and small broadening of Landau levels

$$\mu \gg \tau^{-1} \gg \omega_H, T$$

$$\sigma_{xx} \sim \sigma_{xy} \sim \frac{\mu}{\omega_H}$$

$$z = \frac{\pi}{4}(1 + F_0)\frac{\partial n_e}{\partial \mu}$$

Linear response versus background field formalism

- Linear response

  on trial vector potential $A(q,\omega_n)$

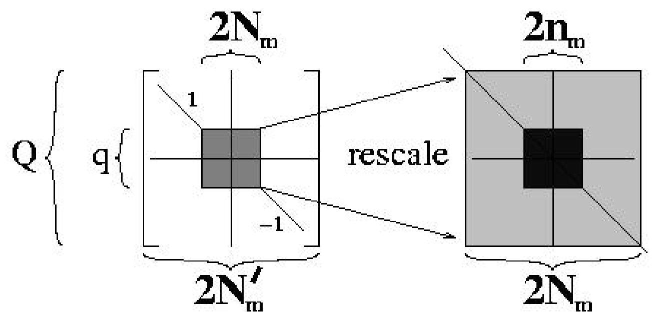
$$\sigma'_{jk}(q,\omega_n) = \epsilon_{jk}\sigma_{xy} - \delta_{jk}\frac{\sigma_{xx}}{4n}\left\langle \mathrm{tr}\,[I_n^\alpha, Q][I_{-n}^\alpha, Q]\right\rangle + \frac{\sigma_{xx}^2}{2nD}\int d\mathbf{r}_2\, e^{i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_2)}\epsilon_{aj}\epsilon_{bk}\langle \mathrm{tr}\, I_n^\alpha Q(\mathbf{r}_1)\nabla_a Q(\mathbf{r}_1) \times \mathrm{tr}\, I_{-n}^\alpha Q(\mathbf{r}_2)\nabla_b Q(\mathbf{r}_2)\rangle$$

- Background fields (A.M. Polyakov, 1975)

  Slow t and fast modes $\quad Q \rightarrow t^{-1}Qt$

  Rescale the size of matrix Q, $\quad N'_m \rightarrow N_m$

  $N_m \rightarrow n_m$



$$\sigma_{jk}^{BF} = \sigma_{jk}^{LR}(q=0,\omega_n\rightarrow 0), \qquad N'_m, N_m \rightarrow \infty$$

- The renormalization of singlet interaction amplitude

$$z' = -\frac{1}{2\pi L^2 \mathrm{tr}\,\eta\Lambda}\frac{\partial \ln Z}{\partial T} = -\frac{z}{2L^2\mathrm{tr}\,\eta\Lambda}\left\langle \int d\mathbf{r}\left[\sum_{\alpha n}\mathrm{tr}\, I_n^\alpha Q\,\mathrm{tr}\, I_{-n}^\alpha Q + 4\,\mathrm{tr}\,\eta Q - 6\,\mathrm{tr}\,\eta\Lambda\right]\right\rangle$$

Calculations

- Expansion in topological sectors

classical + quantum fluctuations

$$\langle O \rangle = \langle O \rangle_0 + \frac{1}{Z_0} \int \mathcal{D}[U_0] \int \frac{d\lambda d\mathbf{r_0}}{\lambda^3} O \, e^{S} + \dots$$

$\mathcal{C}=0$

classical

- Classical values

$$S_\sigma^{\text{cl}} = -2\pi\sigma_{xx} \pm 2\pi i \, \sigma_{xy}, \qquad S_F^{\text{cl}} = \lambda^2 T z \int \frac{d\mathbf{r}}{r^2+\lambda^2} \propto \lambda^2 T z \ln \frac{L^2}{\lambda^2}$$

Logarithmic divergence!

How to define quantum theory?
No constrained instantons!!!
Spatially varying masses (t'Hooft, 1976)

- Quick sketch quantum theory

$$\begin{aligned} S_\sigma^{\text{qf}} &= -2\pi\sigma_{xx}(\lambda) \pm 2\pi i \, \sigma_{xy}(\lambda), \\ S_F^{qf} &= \pi T \lambda^2 z(\lambda) \int_\lambda^\xi \frac{dr}{r} \frac{z(r)}{z(\lambda)} \propto \pi T \lambda^2 z(\lambda) \ln \frac{\xi^2}{\lambda^2} \propto \pi T \lambda^2 z(\lambda) \sigma_{xx}(\lambda) \end{aligned}$$

$$\xi = \lambda \exp(\pi \sigma_{xx}(\lambda)/2)$$

Infrared problems disappear at quantum level!!!

Non-perturbative RG equations

$$\frac{d\sigma_{xx}}{d\ln L} = -\beta_\sigma = -\frac{2}{\pi}\left(1+\frac{1-c}{c}\ln(1-c)\right) - \frac{\beta_2(c)}{\sigma_{xx}} - D(c)\sigma_{xx}^2 e^{-2\pi\sigma_{xx}}\cos 2\pi\sigma_{xy},$$

$$\frac{d\sigma_{xy}}{d\ln L} = -\beta_\theta = -D(c)\sigma_{xx}^2 e^{-2\pi\sigma_{xx}}\sin 2\pi\sigma_{xy},$$

$$\frac{dc}{d\ln L} = -\beta_c = -(1-c)c\left(\frac{1}{\pi\sigma_{xx}}+\frac{\gamma_2(c)}{\sigma_{xx}^2}+D_\gamma(c)\sigma_{xx}e^{-2\pi\sigma_{xx}}\cos 2\pi\sigma_{xy}\right),$$

$$\frac{d\ln z}{d\ln L} = \gamma_z = -c\left(\frac{1}{\pi\sigma_{xx}}+\frac{\gamma_2(c)}{\sigma_{xx}^2}+D_\gamma(c)\sigma_{xx}e^{-2\pi\sigma_{xx}}\cos 2\pi\sigma_{xy}\right)$$

$$D(c) = \frac{4}{\pi}\exp\left[1-4\gamma_E\left(1+\frac{1-c}{c}\ln(1-c)\right)\right] \times \exp\left[-2\frac{1-c}{c}\ln(1-c)[\psi(3-1/c)+\psi(1/c)-1]\right] \times \exp\left[-2\frac{1-c}{c}\left[f(1-1/c)+f(1/c)-2\ln 2\frac{c^2}{2c-1}\right]\right]$$

$$D_\gamma(c) = D(c)\frac{1-c}{2c}\exp\left[\frac{2\ln(1-c)}{c}\right]\int_0^c ds(1-s)^{-2-2/s}$$

$$f(x) = 2x^2\sum_{J=0}^{\infty}\frac{\ln J}{J(J^2-x^2)}$$

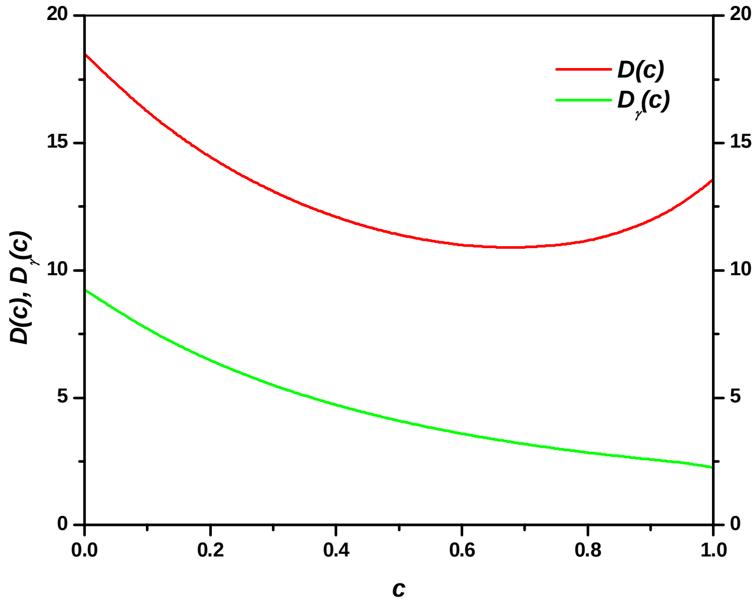


Fig. 6: The instanton determinants $D(c)$ and $D_\gamma(c)$.